R-PARITY VIOLATION AND NEUTRINO MASS

Kayla Hegedus

Submitted in partial fulfulment of the requirements for the degree of Master of Science of Imperial College London

Department of Theoretical Physics
2013

ABSTRACT OF DISSERTATION

Neutrinos offer the first glimpse into physics beyond the Standard Model. The observation of neutrino oscillation leads incontrovertably to the conclusion that neutrinos have mass whereas the Standard Model explicitly forbids it; therefore new theories are necessary. Supersymmetry solves many of the problems in the Standard Model but often is formulated to include R-parity: this Z_2 symmetry differentiates between particles and their supersymmetric partners and is related to the conservation of baryon and lepton number. However, if we allow R-parity to be violated by breaking lepton number, we gain access to additional theories that can account for neutrino masses. Theories with R-parity violation have distinct phenomenological signals that may be observed in the near future.

Kayla Hegedus
Imperial College London
September 2013

Contents

1 Introduction

Though neutrinos are extremely common in our universe, relatively little is known about them. Neutrino oscillation was proposed as a solution to the solar neutrino problem[1] in the 1960s, but it was not until 2003 that this process was experimentally confirmed. This observation has far-reaching consequences: the Standard Model requires that neutrinos are *massless* but the existence of oscillation requires that they are *massive*. One method of giving mass to neutrinos is the seesaw mechanism. In this mechanism, neutrinos must be Majorana particles which is another trait not predicted by the Standard Model. To resolve these contradictions, we must look to theories beyond the Standard Model.

Not only are neutrino masses unexplained, but the Standard Model also, notably, does not include gravity. Unlike the electroweak force which operates at around the TeV scale, gravity is at around 10^{18} GeV. This gigantic discrepency is called the gauge hierarchy problem and is resolved by supersymmetry. Supersymmetry predicts a bosonic partner for every fermion in the Standard Model and a fermionic partner for every Standard Model boson; these new contributions cancel out the problematic ultraviolet terms. We construct the supersymmetric theory with the minimal number of new particles, called the MSSM. While this theory solves many of the problems present in the Standard Model, some new issues arise. Most importantly we have the μ problem, which questions why a supersymmetry-conserving parameter and a supersymmetry-violating one are related. In addition the MSSM, without any restrictions, predicts an unrealistically fast proton decay. To solve this problem, R-parity is implemented. R-parity assigns a quantum number of $+1$ to Standard Model particles and -1 to supersymmetric particles, and by requiring this number to be conserved the problematic interaction terms are forbidden. R-parity can be expressed in terms of baryon and lepton number, specifically the difference $B-L$ between the two. Because of this difference, either baryon or lepton number is allowed to be violated at any given time, which proves to be important for the generation of neutrino mass.

While R-parity is convenient, it is merely an *ad hoc* addition to the MSSM. If we permit R-parity to be violated, new terms are allowed in the Lagrangian that lead to the generation of neutrino mass. These terms can be either bilinear or trilinear, and arise either explicitly or spontaneously. Spontaneous R-parity violation requires adding a new symmetry to the Standard Model gauge group and breaking it, leading to the creation of a new particle. The new terms contribute at the tree and loop level, and several different theories can explain how these contributions lead to accurate neutrino masses and mixings. The collider signatures of these theories are distinct from those of R-parity conserving theories, and may

[1] The observation of one third of the expected number of neutrinos from the sun.

be observed at the upgraded LHC and future projects.

In this paper, I will begin by describing neutrinos in the context of the Standard Model and explaining why they do not have mass. In contrast, I will show how the process of oscillations necessitates neutrino mass, in both the simplified two-flavor and three-flavor mixing cases. Oscillations prove that neutrinos must have mass; this mass can be generated through the seesaw mechanism. I will then motivate and introduce supersymmetry and the minimal supersymmetric Standard Model. With this framework, I discuss R-parity and how it can be violated. Many expansions of the R-parity violating MSSM are proposed in the literature, but I will focus on a representative few to explain how the field and symmetry content of the theory leads to the experimentally correct values for neutrino masses and mixings. Finally, I briefly discuss how these theories may be verified in collider experiments.

2 Neutrinos in the Standard Model

2.1 The Standard Model

The Standard Model describes, to the best of our current knowledge, the most fundamental particle content and interactions that make up the universe. Each element of the Standard Model has been rigorously tested and confirmed, yet various experimental results indicate that it is not fully complete. One such observation is neutrino oscillations, which we shall discuss later. In this section, however, we shall establish the basics of the Standard Model and explain the role of neutrinos within it.

2.1.1 Particle Content

The Standard Model is comprised of three types of interactions– strong, weak and electromagnetic– and the particles that participate in these interactions. Each of these interactions is described by a quantum field theory, which in turn can be represented as gauge theories coupled to fermions [50]. The gauge group for the strong interaction is $SU(3)$, the group for weak interactions is $SU(2)$ and the group for electromagnetic interactions is $U(1)$. The force-carrying bosons in these interactions are associated with the generators of these groups, as summarized in Table 1 [11]. Therefore, the gauge group of the entire Standard Model is:

$$G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y. \tag{1}$$

In this notation, the subscript C refers to "color" which is the quantum number that quarks carry. The subscript L indicates that only left-handed fermions carry

	Strong	**Weak**	**Electromagnetic**
Group	$SU(3)$	$SU(2)$	$U(1)$
Generator	G^{α}_{μ}	W^{α}_{μ}	B_{μ}
number	$\alpha = 1, .., 8$	$a = 1, 2, 3$	1
name	gluon	$W^{\pm}$, Z^0	photon

Table 1: A list of the Standard Model groups with their corresponding generators.

this particular quantum number, and the subscript Y distinguishes between this group which has the weak hypercharge quantum number Y and the different group $U(1)_{EM}$ which has the electrical charge quantum number Q [11].

In Table 1, we see that there are 12 gauge bosons, which are by definition spin-one particles. In addition, the Standard Model contains spin-half particles: the fermions. The six quarks– u, d, c, s, t, b– take part in strong force interactions, while the six leptons– $e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$– do not. The fermions fall into three distinct generations, where each is a more massive copy of the last. Each generation interacts with the gauge bosons in the same way. The fermions of most interest to us will be the neutrinos: ν_e, ν_μ and ν_τ.

In addition to the quantum numbers that fermions possess (Q, color, etc.), they also have a value related to lepton and baryon number. These are simple: each lepton has a value of $n_\ell = +1$ and each antilepton has a value of $n_\ell = -1$; likewise, each quark has a value of $n_q = +1$ and each antiquark has a value of $n_q = -1$ [29]. The n_ℓ contribute to the lepton number, L, which is number of leptons minus the number of antileptons and is conserved in interactions:

$$L = \sum n_\ell. \tag{2}$$

A "quark number" could be defined in exactly the same way, but since we only observe quarks in combination it makes more sense to define a baryon number[2] B:

$$B = \frac{1}{3} \sum n_q. \tag{3}$$

In addition to these conservation laws, lepton *flavor* number is also conserved in the Standard Model. There are three flavor numbers: electron number L_e, muon number L_μ and tau number L_τ; as expected, only the lepton and its neutrino in a certain generation have a value of $L_m = 1$ [29]. These and the other quantum numbers for the fermions are summarized in Table 2. Because there is no lep-

[2] Since mesons are composed of a quark and an antiquark, they have no quark number; meson number therefore is not necessarily conserved in interactions.

	1st generation		**2nd generation**		**3rd generation**	
Quark	u	d	c	s	t	b
Q	$+2/3$	$-1/3$	$+2/3$	$-1/3$	$+2/3$	$-1/3$
n_q	1	1	1	1	1	1
Lepton	e	ν_e	μ	ν_μ	τ	ν_τ
Q	-1	0	-1	0	-1	0
n_q	1	1	1	1	1	1
L_e	1	1	0	0	0	0
L_μ	0	0	1	1	0	0
L_τ	0	0	0	0	1	1

Table 2: A summary of the Standard Model fermions, their electrical charge, and their quark/lepton number. Note that the antifermions have opposite values.

ton flavor mixing in the Standard Model, each of the flavor numbers is conserved independently. Quarks, on the other hand, have flavor mixing according to the Cabbibo-Kobayashi-Maskawa matrix and so have only approximate generational conservation laws [11, 29]. As we will see, the lack of mixing in the lepton sector depends on neutrinos being massless, and so lepton flavor conservation also becomes approximate when neutrino mass is taken into account.

To describe the interactions of the fermions, we must look to their transformation properties in the gauge group of the Standard Model, given in Equation 1 [11]. To do this, we write the spin-half fields as spinors, either Dirac (expressed in lower-case letters, as in Table 2) or, more commonly, Majorana (expressed in capital or script letters).[3] Since each generation interacts in the same way, we indicate this fact by labelling the fields as in Table 3 [11]. We will focus on the lepton sector here: in the Standard Model, leptons are represented by two Majorana fields $E_m(x)$ and $\mathcal{E}_m(x)$. These fields are related to the Dirac spinor $e_m(x)$, which is used to represent the lepton field in quantum electrodynamics, by right- and left-handed projections [11]:

$$e_m(x) = P_L\mathcal{E}_m + P_R E_m. \tag{4}$$

As a result, $\mathcal{E}_m$ is considered the left-handed electron field and E_m the right-handed one. The left-handed field appears in the left-handed projection of an $SU(2)_L$ doublet $L_m(x)$ along with the neutrino field ν_m:

$$P_L L_m(x) = \begin{pmatrix} P_L\nu_m \\ P_L\mathcal{E}_m \end{pmatrix} \tag{5}$$

[3]This notation is the same as in Ref. [11].

	$m=1$	$m=2$	$m=3$
e_m	e	μ	τ
ν_m	ν_e	ν_μ	ν_τ
u_m	u	c	t
d_m	d	s	b

Table 3: Notation for fermion generations.

The right-handed electron field E_m is a singlet with respect to the Standard Model gauge group (as the right-handed neutrino field would be)[4] [11].

Now that the lepton fields are defined, we can describe their transformations. To do this, we give the representations of the gauge group in which the fermion transforms: for $SU(3)_C$ and $SU(2)_L$, the representation is labelled with its dimension, and for $U(1)_Y$ the representation is labelled with the eigenvalue of the generator Y [11]. Therefore, the lepton sector [11]:

$$P_L L_m(x) = \begin{pmatrix} P_L \nu_m \\ P_L \mathcal{E}_m \end{pmatrix} \text{ transforms as } \left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}\right)$$

and

$$P_R E_m \text{ transforms as } (\mathbf{1}, \mathbf{1}, -1)\,.$$

Clearly, the leptons transform in the trivial representation of $SU(3)_C$ as they do not strongly interact. The L_m doublet transforms as a two-dimensional spinor, while the E_m singlet transforms trivially.[5] The right-handed components of the Majorana spinors, $P_R L_m$ and $P_L E_m$, transform in the complex conjugate representations since they are the complex conjugates of the left-handed components [11]. As a result, the Y eigenvalues take on the opposite sign while the other representations remain the same.

2.1.2 The Standard Model Lagrangian

The transformations in the previous section describe the invariance of the Lagrangian under certain symmetries,

$$\delta L_m = \left[\left(-\frac{i}{2}\omega_1(x) + \frac{i}{2}\omega_2^a(x)\tau_a\right) P_L + \left(-\frac{i}{2}\omega_1(x) - \frac{i}{2}\omega_2^a(x)\tau_a^*\right) P_R\right] L_m \qquad (6)$$

[4] Only left-handed helicity neutrinos have been experimentally observed, resulting in the weak force violating parity conservation [55].

[5] If the Standard Model contained a right-handed neutrino field, $P_R N_m$ would transform similarly as $(\mathbf{1}, \mathbf{1}, 0)$.

and

$$\delta E_m = \left[i\omega_1(x)P_L - i\omega_1(x)P_R\right] E_m, \tag{7}$$

where $\omega(x)$ is a scalar function and $T_a = \frac{1}{2}\tau_a$ are the generators of $SU(2)_L$ (τ_a are the 2×2 Pauli matrices) [11]. The full Standard Model Lagrangian that these gauge transformations apply to is

$$\mathcal{L}_{SM} = \mathcal{L}_{int} + \mathcal{L}_{Higgs}, \tag{8}$$

where $\mathcal{L}_{int}$ describes the particle interactions and $\mathcal{L}_{Higgs}$ describes their interactions with the Higgs boson, generating their mass. Explicitly,

$$\begin{aligned}\mathcal{L}_{int} = &-\frac{1}{2}\bar{L}_m \not{D} L_m - \frac{1}{2}\bar{E}_m \not{D} E_m - \frac{1}{2}\bar{Q}_m \not{D} Q_m - \frac{1}{2}\bar{U}_m \not{D} U_m - \frac{1}{2}\bar{D}_m \not{D} D_m \\ &- \frac{1}{4}G^{\alpha}_{\mu\nu}G^{\alpha\mu\nu} - \frac{1}{4}W^{\alpha}_{\mu\nu}W^{\alpha\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &- \frac{g_3^2\Theta_3}{64\pi^2}\epsilon_{\mu\nu\lambda\rho}G^{\alpha\mu\nu}G^{\alpha\lambda\rho} - \frac{g_2^2\Theta_2}{64\pi^2}\epsilon_{\mu\nu\lambda\rho}W^{\alpha\mu\nu}W^{\alpha\lambda\rho} - \frac{g_1^2\Theta_1}{64\pi^2}\epsilon_{\mu\nu\lambda\rho}B^{\mu\nu}B^{\lambda\rho},\end{aligned} \tag{9}$$

where $G^{\alpha}_{\mu\nu}$, $W^{\alpha}_{\mu\nu}$ and $B_{\mu\nu}$ are the gauge field strengths and U_m, D_m and Q_m are the quark fields[6] [11]. In this Lagrangian, only terms that are singlets under the Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ can appear or else the expression is not gauge invariant; therefore, *no mass terms can appear* [11]. To get terms for particle mass, this symmetry must be spontaneously broken, which we do by introducing the Higgs field

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \tag{10}$$

which transforms as $(\mathbf{1}, \mathbf{2}, +1/2)$ [11]. As a result, we get the $\mathcal{L}_{Higgs}$ terms:

$$\begin{aligned}\mathcal{L}_{Higgs} = &-\left(D_\mu\phi\right)^\dagger\left(D^\mu\phi\right) - V\left(\phi^\dagger\phi\right) \\ &- \left(f_{mn}\bar{L}_m P_R E_n \phi + h_{mn}\bar{Q}_m P_R D_n \phi + g_{mn}\bar{Q}_m P_R U_n \tilde{\phi}\right)\end{aligned} \tag{11}$$

where $\tilde{\phi}$ is the complex conjugate of ϕ [11]. The scalar potential $V\left(\phi^\dagger\phi\right)$ takes the form

$$V\left(\phi^\dagger\phi\right) = \lambda\left(\phi^\dagger\phi\right)^2 - \mu^2\phi^\dagger\phi + \mu^4/4\lambda; \tag{12}$$

λ and μ^2 are real and positive [11].

---

[6] Similar to L_m, $P_L Q_m = \begin{pmatrix} P_L \mathcal{U}_m \\ P_L \mathcal{D}_m \end{pmatrix}$.

2.1.3 Particle Masses

The general Lagrangian in Equation 8 cannot contain mass terms, yet we know experimentally that many of the Standard Model particles are massive. In order to see these masses in the Lagrangian, we specify the gauge to be the unitary gauge, defined by

$$\phi = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + H(x)) \end{pmatrix}, \tag{13}$$

where $H(x)$ is a real field and v is the vacuum expectation value (v.e.v.)[7] found by minimizing the potential in Equation 12: $v = \mu^2/\lambda$.

The vacuum expectation value describes the ground state of the Standard Model system, and so we perturb the Lagrangian around this state by expanding the Lagrangian in Equation 8 in terms of v. The terms in $\mathcal{L}_{int}$ expand to be the kinetic terms in the free Lagrangian $\mathcal{L}_0$, so we focus our attention on $\mathcal{L}_{Higgs}$, as expected. Expanding the leptonic term, we get

$$\begin{aligned} f_{mn}\bar{L}_m P_R E_n \phi &= f_{mn} \begin{pmatrix} \bar{\nu}_m \\ \bar{\mathcal{E}}_m \end{pmatrix}^\top P_R E_n \phi \\ &= f_{mn} \begin{pmatrix} \bar{\nu}_m & \bar{\mathcal{E}}_m \end{pmatrix} P_R E_n \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + H) \end{pmatrix} \\ &= \frac{1}{\sqrt{2}} f_{mn} (v + H) \bar{\mathcal{E}}_m P_R E_n. \end{aligned} \tag{14}$$

The quark terms are expanded in a similar way. As a result, we get the following relevant terms in $\mathcal{L}_{Higgs}$:

$$\mathcal{L}_v = -\frac{v}{\sqrt{2}} \left(f_{mn}\bar{\mathcal{E}}_m P_R E_n + g_{mn}\bar{\mathcal{U}}_m P_R U_n + h_{mn}\bar{\mathcal{D}}_m P_R D_n + \text{h.c.} \right) \tag{15}$$

where h.c. represents the hermitian conjugate terms [11]. These mass terms are not necessarily diagonal in the generation indices but we can transform the fields so that their mass terms are diagonal, making each particle's mass easy to determine [11]. The fields are redefined as

$$P_L\mathcal{E}_m = U_{mn} P_L \mathcal{E}'_n \tag{16}$$

$$P_L E_m = V_{mn} P_L E'_n \tag{17}$$

where U_{mn} and V_{mn} are matrices that "mix" the generations [11]. The new term

[7]Note that v is *not* zero so that the $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry is broken.

for the leptons in the Lagrangian is thus

$$\mathcal{L}_{Higgs} = (U^{\top}_{mn} f_{mn} V_{mn}) \bar{\mathcal{E}}'_m P_R E'_n. \tag{18}$$

The matrices U_{mn} and V_{mn} are general, so we can choose them so that $U_{mn} = V^*_{mn}$ and specify U_{mn} so that the mass matrix f_{mn} is diagonalized [11]:

$$U^{\top}_{mn} f_{mn} V_{mn} = V^{\top}_{mn} f_{mn} V_{mn} = \mathrm{diag}(f_e, f_\mu, f_\tau) = f_m. \tag{19}$$

Rewriting Equation 15 and omitting the primes, the fermion mass terms in the Lagrangian are now

$$\mathcal{L}_v = -\frac{v}{\sqrt{2}} \left(f_m \bar{\mathcal{E}}_m P_R E_m + g_m \bar{\mathcal{U}}_m P_R U_m + h_m \bar{\mathcal{D}}_m P_R D_m + \text{h.c.} \right); \tag{20}$$

note that the quark terms transform in the same way as the leptons.

We can now write the fields in terms of the QED Dirac spinor fields, as in Equation 4. In the Lagrangian, we have the terms

$$\begin{aligned} \bar{\mathcal{E}}_m P_R E_m + \text{h.c.} &= \bar{\mathcal{E}}_m P_R E_m + \bar{\mathcal{E}}_m P_L E_m \\ &= \bar{\mathcal{E}}_m P_R E_m + \bar{E}_m P_L \mathcal{E}_m \\ &= \bar{e}_m P_R E_m + \bar{e}_m P_L e_m \\ &= \bar{e}_m e_m \end{aligned} \tag{21}$$

and so the Lagrangian mass terms are finally in a useful form [11]:

$$\mathcal{L}_v = -\frac{v}{\sqrt{2}} \left(f_m \bar{e}_m e_m + g_m \bar{u}_m u_m + h_m \bar{d}_m d_m \right). \tag{22}$$

Therefore, we now have equations for the masses of the leptons (and quarks),

$$m_e = \frac{v}{\sqrt{2}} f_e \tag{23}$$

$$m_\mu = \frac{v}{\sqrt{2}} f_\mu \tag{24}$$

$$m_\tau = \frac{v}{\sqrt{2}} f_\tau, \tag{25}$$

where each f_m is completely independent of the others.

The most relevant result for us is that there is *no* term for the neutrinos, meaning that, in the Standard Model, neutrinos are massless. This result could have been anticipated in Equation 14, where the 0 term in the unitary gauge negated the ν term, thus preventing the neutrino sector from interacting with the

Higgs field and gaining a mass.

2.1.4 Interactions

We will briefly examine the interactions between the fermions and other particles to complete our discussion of the Standard Model.

The fermion's coupling to the Higgs boson is the simplest to analyze since we already accomplished much of the work in the previous section. Recall that we began with Equation 14 and had two important terms:

$$\frac{v}{\sqrt{2}} f_{mn} \bar{\mathcal{E}}_m P_R E_n \tag{26}$$

and

$$\frac{H}{\sqrt{2}} f_{mn} \bar{\mathcal{E}}_m P_R E_n. \tag{27}$$

The vacuum expectation value v term (Equation 26) determined the mass terms for the fermions, and the Higgs field H term (Equation 27) determines the interaction between the fermions and the Higgs boson. Following the same procedure as in the previous section, we transform the fields so that the interaction terms have the following form:

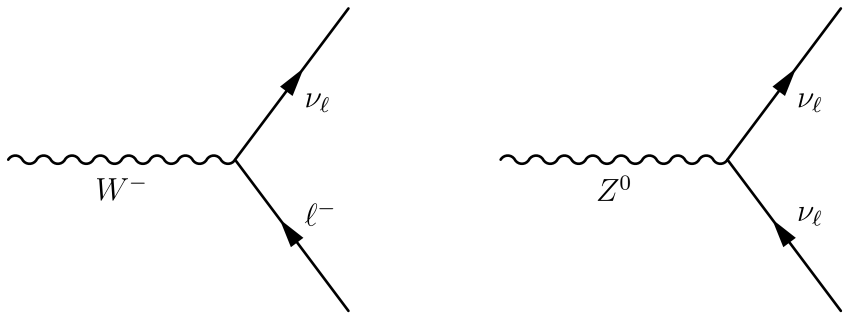
$$\begin{aligned}\mathcal{L}_{Hf} &= -\frac{H}{\sqrt{2}}\left(f_m \bar{e}_m e_m + g_m \bar{u}_m u_m + h_m \bar{d}_m d_m\right) \\ &= -\sum \frac{m_f}{v} \bar{f} f H,\end{aligned} \tag{28}$$

where we are summing over all fermions f (of course, m_ν is zero). The vacuum expectation value is $v = 246$ GeV and so $m \ll v$ for the fermions [11]. Therefore, fermions couple weakly to the Higgs boson: the heavier the fermion is, the more strongly they couple.

Since leptons by definition do not interact in the strong force, the only other interactions to describe are electroweak. Electroweak interactions come in two varieties: charged-current and neutral-current. Charged-current interactions arise from couplings with W_μ^α and neutral-current interactions arise from couplings with B_μ, as in these kinetic terms in the Lagrangian in Equation 8:

$$\mathcal{L}_{EW} = -\frac{1}{2}\bar{L}_m \not{D} L_m - \frac{1}{2}\bar{E}_m \not{D} E_m - \frac{1}{2}\bar{Q}_m \not{D} Q_m - \frac{1}{2}\bar{U}_m \not{D} U_m - \frac{1}{2}\bar{D}_m \not{D} D_m. \tag{29}$$

We expand the leptonic part of this equation in terms of the mass eigenstates and



(a) A charged-current interaction. (b) A neutral-current interaction.

Figure 1: The two basic vertices of weak interactions.

the covariant derivative:

$$\begin{aligned}\mathcal{L}_{EW} = &+ \frac{i}{4}\begin{pmatrix}\bar{\nu}_m \\ \bar{\mathcal{E}}_m\end{pmatrix}^{\top}\gamma_\mu P_L \begin{pmatrix}-g_1 B_\mu + g_2 W^3_\mu & g_2(W^1_\mu - iW^2\mu) \\ g_2(W^1_\mu + iW^2\mu) & -g_1 B_\mu - g_2 W^3_\mu\end{pmatrix}\begin{pmatrix}\nu_m \\ \mathcal{E}_m\end{pmatrix} \\ &- \frac{i}{2} g_1 B_\mu \bar{E}_m \gamma^\mu P_R E_m + \text{h.c.};\end{aligned} \tag{30}$$

the first line is the charged-current interactions and the second line is the neutral-current interactions as pictured in Figures 1a and 1b [11].

The charged-current interaction terms involve a projection operator, so we can rewrite them in terms of the Dirac spinor e_m:

$$\mathcal{L}_{cc} = \frac{ig_2}{\sqrt{2}}\left(W^+_\mu(\bar{\nu}_m\gamma^\mu P_L e_m) + W^-_\mu(\bar{e}_m\gamma^\mu P_L \nu_m)\right), \tag{31}$$

where $W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu)$ [11]. We want to transform the Dirac fields into the mass basis as in Equation 17. Though neutrinos are massless, we can still transform them as

$$\nu_m = U^e_{mn}\nu'_m. \tag{32}$$

We transform the quark sector in the same way and define $V_{mn} = (U^{u\dagger}U^d)_{mn}$; with these definitions, the charged-current terms are [11]

$$\begin{aligned}\mathcal{L}_{cc} = &\frac{ig_2}{2\sqrt{2}}\left[W^+_\mu(\bar{\nu}'_m\gamma^\mu(1+\gamma_5)e'_m) + W^-_\mu(\bar{e}'_m\gamma^\mu(1+\gamma_5)\nu'_m)\right. \\ &\left.+V_{mn}(\bar{u}'_m\gamma^\mu(1+\gamma_5)d'_m) + V^\dagger_{mn}(\bar{d}'_m\gamma^\mu(1+\gamma_5)u'_m)\right].\end{aligned} \tag{33}$$

The 3×3 unitary matrix V_{mn} is the Cabbibo-Koyabashi-Maskawa (CKM) matrix and enables charged-current interactions to happen between quark generations [11]. This matrix is nearly, but not exactly, a unit matrix so these cross-generational interactions are rare. Because neutrinos are massless there is no equivalent matrix for the lepton sector in the Standard Model.

The neutral-current interactions act in the same way for all fermions. The electroweak gauge group has two bosons, A_μ and Z_μ, that are related to the generators in this way:

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \tag{34}$$

where θ_w is the weak mixing angle [11]. After some rearranging, we get two terms defining the fermion coupling to the photon A_μ and the Z-boson Z_μ. The photon or electromagnetic coupling is

$$\mathcal{L}_{em} = \sum_f ieA_\mu \bar{f}\gamma^\mu Q f \tag{35}$$

and the Z-boson or neutral-current coupling is

$$\mathcal{L}_{nc} = \frac{ie}{\sin\theta_w \cos\theta_w} \sum_f Z_\mu \bar{f}\gamma^\mu (g_V + \gamma_5 g_A) f, \tag{36}$$

where g_V and g_A are constants given by various generators [11]. We note that since neutrinos have electric charge $Q = 0$, they only participate in neutral-current interactions.

2.2 Neutrino Oscillations

2.2.1 Motivation

The Standard Model has been remarkably successful experimentally, but the discovery of neutrino oscillations has presented a major challenge.[8] The first hints of an issue came with the solar neutrino problem: in 1939 Hans Bethe discovered the reaction that occurs in the interior of the Sun, the *pp* chain [16]. This process went through several theoretical iterations, but the one that occurs in most of the Sun's reactions, and, more importantly to us, produces neutrinos is as follows [29]:

The pp *chain.* To begin, two protons, or hydrogen nuclei, combine to form a deuteron:

$$\begin{aligned} p + p &\to d + e^+ + \nu_e \\ p + p + e^- &\to d + \nu_e. \end{aligned}$$

[8] Section 2.2 is taken, in large parts, from Ref. [31].

This deuteron then combines with another proton:

$$d + p \to {}^3\text{He} + \gamma.$$

Helium-3 creates alpha particles or beryllium-7:

$$\begin{aligned} {}^3\text{He} + p &\to \alpha + e^+ + \nu_e \\ {}^3\text{He} + {}^3\text{He} &\to \alpha + p + p \\ {}^3\text{He} + \alpha &\to {}^7\text{Be} + \gamma. \end{aligned}$$

This beryllium-7 converts into alpha particles in two ways, either:

$$\begin{aligned} {}^7\text{Be} + e^- &\to {}^7\text{Li} + \nu_e \\ {}^7\text{Li} + p &\to \alpha + \alpha \end{aligned}$$

or:

$$\begin{aligned} {}^7\text{Be} + p &\to {}^8\text{B} + \gamma \\ {}^8\text{B} &\to {}^7\text{Be} + e^+ + \nu_e \\ {}^7\text{Be} &\to \alpha + \alpha. \end{aligned}$$

Note that electron-neutrinos are produced in several of these steps.

Because neutrinos interact so weakly with other particles these solar neutrinos offer a probe into the Sun's interior, a fact that inspired Ray Davis to look specifically for them in 1966 [16]. However, his experiments only detected a rate of 2.2 ± 0.2 SNU, a rate about one third of the expected 8 SNU[9] [5]. Pontecorvo and Gribov proposed a theoretical solution: more than one type of neutrino existed (at this point, only the electron-neutrino was known) and they could transform into one another in a process called oscillation [16]. This prediction was eventually validated at the Sudbury Neutrino Observatory, which detected all three types of neutrinos and observed in 2003 that the sum of all three flavours corresponded to the original predicted solar neutrino rate [16]. As only electron-neutrinos are produced in the *pp* chain, this result gave strong evidence for the existence of neutrino oscillations.

[9] SNU stands for "solar neutrino unit" and is equal to 10^{-36} interactions per second; for 1 SNU, one detection would occur about every six days [5].

2.2.2 Two-Flavor Model

Though there are three generations of neutrinos, a simplified two-flavor model gives us the important features and results of oscillation. In this model, we consider only ν_e and ν_μ. These are the weak eigenstates ν_α that we detect in weak interactions; however, we must also consider the mass/energy eigenstates ν_i that are the eigenstates of free Hamiltonian of the system.

The inequivalence of the weak and mass eigenstates allow neutrino oscillations to occur. The weak eigenstates are a linear combination of the mass eigenstates and are related by a mixing matrix:

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \nu_e \end{pmatrix}. \tag{37}$$

The angle θ is called the mixing angle and measures the amount of mixing that occurs between the mass eigenstates. This amount changes in time so that at certain moments the system is most similar to ν_e and at other moments is most similar to ν_μ. Thus, the probability of observing a given state at any point in time changes, and since the behaviour is regulated by the periodic functions sine and cosine we say that it oscillates. In the simplified two-flavor system, the mixing matrix is the usual rotation matrix and so has only one mixing angle, but in the three-flavor system the mixing matrix will be 3×3 and contain three different mixing angles– θ_{12}, θ_{13} and θ_{23}– controlling the mixing between each pair of neutrino generations.

To specify exactly how the mass eigenstates change in time, we must look at the time-dependent Schrödinger equation for the system. We will define an operator that changes the initial state as a function of time [10]:

$$|\psi(t)\rangle = \hat{U}(t)|\psi(0)\rangle. \tag{38}$$

To solve for $\hat{U}(t)$, we find the Schrödinger equation for it, which is

$$\frac{d}{dt}\hat{U}(t) = -\frac{i}{\hbar}\hat{H}\hat{U}(t), \tag{39}$$

and solving this equation for $\hat{U}(t)$ (assuming that $\hat{H}$ is time-independent and U is unitary) yields

$$\hat{U}(t) = e^{-i\hat{H}t/\hbar}. \tag{40}$$

Therefore, we can rewrite our Equation 38 as

$$|\psi(t)\rangle = e^{-i\hat{H}t/\hbar}|\psi(0)\rangle. \tag{41}$$

The original states ν_i are eigenstates of $\hat{H}$ so we can replace $\hat{H}$ with E_i. These Es are the allowed energies of the system and thus are the eigenvalues associated with the eigenstates [10]. As a result, we can write the time-dependent mass eigenstates as

$$\nu_1(t) = \nu_1(0)e^{-iE_1t/\hbar} \tag{42}$$

$$\nu_2(t) = \nu_2(0)e^{-iE_2t/\hbar}. \tag{43}$$

With these expressions, we can now rewrite Equation 37 in the time-dependent form:

$$\begin{pmatrix} \nu_1(0)e^{-iE_1t/\hbar} \\ \nu_2(0)e^{-iE_2t/\hbar} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_\mu(t) \\ \nu_e(t) \end{pmatrix}. \tag{44}$$

We are most interested in the weak eigenstates as they are the ones observed in weak scattering events. In order to solve for the weak eigenstates in terms of the mass eigenstates, we must find the inverse of the rotation matrix in Equation 44, which fortunately is simply a matter of moving the negative sign. The time-dependent equations for the weak eigenstates is thus

$$\begin{pmatrix} \nu_\mu(t) \\ \nu_e(t) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1(0)e^{-iE_1t/\hbar} \\ \nu_2(0)e^{-iE_2t/\hbar} \end{pmatrix}. \tag{45}$$

Once we have this equation, we must determine the values of $\nu_1(0)$ and $\nu_2(0)$. These states depend on what type of neutrino existed at t=0; that is, which flavor of neutrino was produced in the original weak interaction. Say an electron-neutrino was produced (as occurs in the Sun): at t=0, then, ν_e=1 and ν_μ=0. Putting these values into Equation 44, we determine that $\nu_1(0)$=$-\sin\theta$ and $\nu_2(0)$=$\cos\theta$. We then put these values into Equation 45 in order to find the full time-dependent equations for the weak eigenstates:

$$\nu_\mu(t) = \sin\theta\cos\theta(e^{-iE_2t/\hbar} - e^{-iE_1t/\hbar}) \tag{46}$$

$$\nu_e(t) = \sin^2\theta e^{-iE_2t/\hbar} + \cos^2\theta e^{-iE_1t/\hbar}. \tag{47}$$

Now, to find the probability that the original electron-neutrino has transformed into a muon-neutrino, we calculate $|\nu_\mu|^2$. After some algebra, we find that

$$P_{\nu_e\to\nu_\mu} = \sin^2 2\theta \sin^2\left(\frac{E_2 - E_1}{2\hbar}t\right). \tag{48}$$

To find the probability that it has remained ν_e, we simply subtract the probability in Equation 48 from 1; this value is called the survival probability. Using the equation $E^2 = |\vec{p}|^2c^2 + (mc^2)^2$ in the highly relativistic limit and finding m, the mass associated with the mass eigenstates, we can rewrite Equation 48 as

$$P_{\nu_e \to \nu_\mu} = \sin^2 2\theta \sin^2 \left(\frac{m_2^2 - m_1^2}{4\hbar E} c^4 t \right), \tag{49}$$

where E is the energy of the emitted neutrino [29]. Assuming that neutrinos travel at approximately the speed of light and writing the difference of the masses squared as Δm^2, the oscillation probability in vacuum is

$$P_{\nu_e \to \nu_\mu} = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 c^3}{4\hbar E} L \right), \tag{50}$$

where L is the distance the neutrino has travelled [29].

From this admittedly simplified equation, we can still see the basic form and consequences of neutrino oscillation. First, the probability that an electron-neutrino will transform into a muon-neutrino is periodic, related to both the mixing angle and the difference in masses associated with the mass eigenstates. This periodicity implies that, once transformed, the muon-neutrino will eventually change back into its original state, hence the name "neutrino oscillations." In addition, the fact that oscillations depend on the mass difference indicates that these masses must be different: neutrinos *must be massive* and have nonequal masses. Lastly, we note that lepton flavor number, as in Table 2, is no longer conserved in weak interactions, as any of the three neutrinos may be detected in any given interaction.

2.2.3 Three-Flavor Model

With the addition of the third flavor ν_τ to our model, the mathematics become less transparent. The weak states are still linear combinations of the mass eignestates, but we represent this more generally:

$$|\nu_\alpha\rangle = \sum_{i=1}^{n} U^*_{\alpha i} |\nu_i\rangle \tag{51}$$

where n is the number of neutrino generations (which is assumed, from experimental evidence, to be three) and U is the mixing matrix [28]. The mixing matrix U is unitary and has several possible parameterizations, such as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) or the Maki-Nakagawa-Sakata (MNS) mixing matrix [44]. The MNS matrix takes the form

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{-i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{-i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{-i\delta} & -c_{12}s_{23} - s_{12}c_{23}c_{13}e^{-i\delta} & c_{23}c_{13} \end{pmatrix} \tag{52}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ [29]. Instead of one mixing angle as in the two-flavor case, we have three mixing angles and a phase. These phases differentiate the MNS matrix from the CKM matrix for quarks: in the CKM matrix any phases can be rotated away since the Standard Model Lagrangian conserves flavor and is thus unaffected by this transformation [11]. In the MNS matrix, though, we cannot perform the same trick because lepton number is clearly not conserved in neutrino oscillations.

After a time t, ν_α evolves, predictably, as

$$|\nu_\alpha(t)\rangle = \sum_{i=1}^{n} U^*_{\alpha i}|\nu_i(t)\rangle. \tag{53}$$

The probability of detecting the state $|\nu_\beta\rangle$ can be written as

$$P_{\alpha\beta} = |\langle\nu_\beta|\nu_\alpha(t)\rangle| = \Big|\sum_{i=1}^{n}\sum_{j=1}^{n} U^*_{\alpha i}U_{\beta j}\langle\nu_\beta|\nu_\alpha(t)\rangle\Big|^2 \tag{54}$$

or, if we look at only the amplitude [44],

$$A_{\alpha\beta} = \sum_{i=1}^{n} U_{\beta i}D_i U^\dagger_{i\alpha}. \tag{55}$$

In this case, D_i is a function of momentum, the distance traveled and time, and so describes the propagation of ν_i over the distance L [44]. Using relativistic quantum mechanics (and setting $\hbar = c = 1$), we find that

$$D_i = e^{-i(E_iT - p_iL)} \tag{56}$$

where T is the propagation time and L is the distance travelled by the neutrino [44].

In order to calculate the transition probability $P_{\alpha\beta}$, we square Equation 55. From the factor $D_iD_j^*$ we get the phase [44]

$$\begin{aligned}\delta\phi_{ij} &= (E_i - E_j)T - (p_i - p_j)L \\ &= (E_i - E_j)\left(T - \frac{E_i + E_j}{p_i + p_j}L\right) + \frac{m_i^2 - m_j^2}{p_i + p_j}L. \end{aligned}\tag{57}$$

This equation can be simplified by approximating $L{=}T$, which can be justified in three distinct ways. First, T and L can be related by $T{=}(E_i + E_j)L/p_i + p_j{=}L\bar{v}$, where $\bar{v}$ is the average velocity of ν_i and ν_j. In addition, the same result occurs if we assume that $E_i{=}E_j{=}E_0$. Finally, we can say that $p_i{=}p_j{=}p$ and $L{=}T$ up to terms m_{ij}^2/p^2 for relativistic neutrinos; alternately, if $E_i \neq E_j$ and $p_i \neq p_j$ and

$L{=}T$ up to terms of order $m_{i,j}^2/E_{i,j}^2$ for relativistic neutrinos, we reach the same conclusion [44]. For all three of these situations, we conclude that Equation 57 simplifies to

$$\delta\phi_{ij} \simeq \frac{m_i^2 - m_j^2}{2p} L = 2\pi \frac{L}{L_{ij}^{\nu}} \mathrm{sgn}(m_i^2 - m_j^2) \tag{58}$$

where p is the average of p_i and p_j and L_{ij}^{ν} is the neutrino oscillation length associated with the mass difference between ν_i and ν_j [44]:

$$L_{ij}^{\nu} = \frac{4\pi L}{\Delta m_{ij}^2}.$$

This value is useful in setting up neutrino oscillation experiments, as the distance L between the source and the detector must be of the same order or greater than L_{ij}^{ν} in order for the oscillations to be detected.

With Equations 55, 57 and 58, we find the transition probability to be

$$P_{\alpha\beta} = \sum_i |U_{\beta i}|^2 |U_{\alpha i}|^2 + 2 \sum_{i>j} |U_{\beta i} U_{\alpha i}^* U_{\alpha j} U_{\beta j}^*| \cos\left(\frac{\Delta m_{ij}^2}{2p} L + \phi_{\alpha\beta;ij}\right) \tag{59}$$

where $\phi_{\alpha\beta;ij}{=}\arg(U_{\beta i} U_{\alpha i}^* U_{\alpha j} U_{\beta j}^*)$ [44].

Though this probability is more complex than Equation 50, the same Δm_{ij}^2 term appears in both equations and so indicates that all three neutrinos have unique, non-zero masses. The mass difference has been measured to be on the order of $\Delta m_{ij} \leq 5 \times 10^{-3}$ eV^2, and so require a large L and small E to probe closely [11]. We can determine the mass differences in this way, but the actual masses of the neutrinos remain unspecified.

3 Beyond the Standard Model

Since the current Standard Model theory predicts massless neutrinos while the observation of neutrino oscillations indicates that neutrinos do have mass, we must clearly must look beyond the Standard Model to find an explanation for this fact. This section will explore some mechanisms for neutrino mass and describe the basic supersymmetric theory before we consider R-parity in more depth.

3.1 The Seesaw Mechanism

3.1.1 Majorana Neutrinos

One of the simplest methods for modifying the Standard Model is the addition of new particles, so we introduce sterile neutrinos to the theory. These neutrinos are "sterile" as they do not interact with any of the existing Standard Model particles, but they do have mass. As their presence does not affect current observables, we are free to add as many sterile neutrinos as required [28]. Since they are non-interacting, the only terms sterile neutrinos add to the Standard Model Lagrangian are mass terms. These terms come in two varieties: Dirac and Majorana[10] [38]. The Dirac mass term for neutrinos has the form

$$-m_D\left(\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L\right) \tag{60}$$

and the Majorana mass term has the form

$$-\frac{1}{2}m_M^L\left(\bar{\nu}_L\nu_L^c + \bar{\nu}_L^c\nu_L\right) - \frac{1}{2}m_M^R\left(\bar{\nu}_R\nu_R^c + \bar{\nu}_R^c\nu_R\right), \tag{61}$$

where L and R denote left- and right-handed neutrinos and c denotes a charge conjugated neutrino [38]. The $m \times 3$ matrix m_D, where m is the number of sterile neutrinos, is generated from the Higgs spontaneous symmetry breaking and so has a form analagous to those in Equation 22:

$$m_D = \frac{v}{\sqrt{2}}j_\nu \tag{62}$$

where v is the Higgs vacuum expectation value and j_ν is the mass matrix [28]. The $m \times m$ matrix m_M is symmetric and only permitted in the theory if the neutrinos are sterile: the neutrinos cannot have an electric, flavor or color charge, thus preventing them from interacting via Standard Model forces [28].

The Dirac and Majorana mass terms in Equations 60 and 61 can be combined into a single term [28]:

$$-\mathcal{L}_{mass} = \frac{1}{2}\bar{\vec{\nu}}^c M_\nu \vec{\nu} + \text{h.c.}. \tag{63}$$

[10] These names do not refer to the spinor representation of the fields in these terms nor the type of particle described.

The vectors take the forms [38]

$$\vec{\bar{\nu}}^c = (\bar{\nu}_L \ \bar{\nu}_R^c) \tag{64}$$

$$\vec{\nu} = \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}. \tag{65}$$

The $(3+m) \times (3+m)$ symmetric matrix M_ν takes the form

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix} \tag{66}$$

where the first term is zero because we are only considering left-handed neutrinos [38]. Using the unitary matrix V^ν we can diagonalize M_ν and so easily find the $3+m$ mass eigenstates $\vec{\nu}_{mass} = (V^\nu)^\dagger \vec{\nu}$ [28]. Using these values, the Lagrangian can be rewritten as

$$-\mathcal{L}_{mass} = \frac{1}{2} \sum_{k=1}^{3+m} m_k \bar{\nu}_{kM} \nu_{kM} \tag{67}$$

where k counts the neutrinos and M indicates that we are now considering the neutrinos in the mass, not weak, basis [28]. The neutrino terms are the sum of two mass terms:

$$\nu_{Mk} = \nu_{k,mass} + \nu_{k,mass}^c; \tag{68}$$

importantly, we see that $\nu_M = \nu_M^c$ [28]. This result is the Majorana condition. The mass states of neutrinos and antineutrinos are equivalent and so both can be described by a single field; that is, neutrinos in this model are Majorana particles.

The fact that neutrinos are Majorana particles has several important implications, the first being the violation of total lepton number [55]. Recall that in the Standard Model, neutrinos and charged leptons have a lepton number of $+1$; likewise, antineutrinos and antileptons have a lepton number of -1. However, if $\nu = \nu^c$, then because charge conjugation reverses lepton number we cannot consistently assign a lepton number to the neutrino; i.e, a Majorana neutrino's lepton number must be 0. Even with this change, though, lepton number cannot be conserved: in the interaction $u + \bar{d} \to e^+ + \nu_{eM}$, for example, the lepton number of the left-hand side is 0 while on the right-hand side it is -1.

Majorana neutrinos have a clear experimental indicator: neutrinoless double beta decay [29]. In this interaction, a nucleus emits an electron and neutrino but this neutrino is absorbed as an *antineutrino* into a second nucleus, which then emits an electron, as pictured in Figure 2 [55]. The observation of this decay would support the existence of Majorana neutrinos, but could also be indicative of a variety of theories such as left-right symmetric GUTs or, more interestingly for us, R-parity violating supersymmetry [24, 34]. In any case, its observation would

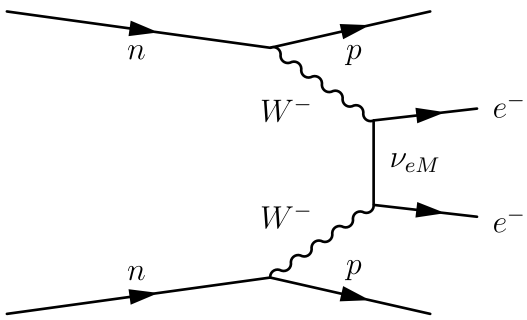


Figure 2: Double beta decay, where ν_{eM} is a Majorana electron-neutrino.

give solid evidence for physics beyond the Standard Model.

3.1.2 The Seesaw Mechanism and Neutrinos

Now that we have a theory for the Majorana nature of neutrinos, we address the question of neutrino mass.[11] The key questions are: why do they have mass and why is this mass so much less than that of the other Standard Model particles [38]? We will assume that $m_N \gg m_D$, which means that the masses generated by the Majorana term of the Lagrangian are much greater than the Dirac term masses. As a result of this choice, the matrix M_ν in Equation 66 has the form

$$M_\nu = \begin{pmatrix} 0 & \simeq 0 \\ \simeq 0 & m_N \end{pmatrix}. \tag{69}$$

Diagonalizing this matrix results in three light neutrinos ν_l, and m heavy neutrinos N so that the Lagrangian is [28]:

$$-\mathcal{L}_{M_\nu} = \frac{1}{2}(\bar{\nu}_l M_{light} \nu_l + \bar{N} M_{heavy} N). \tag{70}$$

These light and heavy masses are related to the Majorana and Dirac masses as

$$M_{light} = -m_D m_N^{-1} m_D^\top \tag{71}$$

and

$$M_{heavy} = m_N. \tag{72}$$

Equation 71 is known as the Type I seesaw formula [8]. As the name suggests, several types of seesaw mechanisms exist: the Type II seesaw mechanism adds a Higgs triplet that couples to both Majorana and Dirac neutrinos as well as the conventional Higgs, while the Type III seesaw works in many unified theories and

[11] This section is primarily from Ref. [31].

may more accurately explain neutrino mixing and leptogenesis [8]. In general, the seesaw mechanism can be implemented in one of these three versions, with different gauge groups and multiplets, broken explictly or spontaneously, at a high or low energy scale, and with or without supersymmetry [56]. Within this great variety, the basics remain the same: the seesaw mechanism is so named because the light and heavy masses are inversely proportional, so that as the heavy mass becomes heavier, the lighter mass becomes lighter. Both of these light and heavy neutrinos are Majorana particles [28].

If we rewrite Equation 66, while no longer ignoring right-handed neutrinos, as

$$M_\nu = \begin{pmatrix} m_N^R & m_D^\top \\ m_D & m_N^L \end{pmatrix} \tag{73}$$

and solve it for its eigenvalues and eigenstates, we will find that the heavy neutrinos are right-handed and the light neutrinos are left-handed [28, 38]. The two eigenvalues can be easily found [38]:

$$\lambda_{1,2} = \frac{1}{2}\left((m_N^R + m_N^L) \pm \sqrt{(m_N^R + m_N^L)^2 - 4(m_N^R m_N^L - m_D^2)}\right). \tag{74}$$

Now, we can rewrite Equation 73 in the mass (instead of weak) basis, where only the Majorana mass term M_N couples to the Higgs:

$$M_m = \begin{pmatrix} m_\nu & 0 \\ 0 & M \end{pmatrix}. \tag{75}$$

The eigenstates, ν and N, are the mass eigenstates for the neutrinos [38]. We will assume that the Higgs field couples very weakly to the ν-field so that $m_\nu \simeq 0$ [38]. This clearly means that m_ν is the light mass and that λ_1 in the weak basis must also be negligible:

$$\begin{aligned} \lambda_1 = 0 &= \frac{1}{2}\left((m_N^R + m_N^L) \pm \sqrt{(m_N^R + m_N^L)^2 - 4(m_N^R m_N^L - m_D^2)}\right) \\ 0 &= (m_N^R + m_N^L) \pm \sqrt{(m_N^R + m_N^L)^2 - 4(m_N^R m_N^L - m_D^2)} \\ (m_N^R + m_N^L)^2 &= (m_N^R + m_N^L)^2 - 4(m_N^R m_N^L - m_D^2) \\ 0 &= -4(m_N^R m_N^L - m_D^2) \end{aligned} \tag{76}$$

resulting in

$$m_N^R m_N^L = m_D^2. \tag{77}$$

This equation also reveals the nature of the seesaw mechanism: for a fixed M_D, the

Majorana masses must be inversely proportional, which is exactly what we found in Equation 71 [38]. Note also that we chose the negative sign for λ_1; therefore, $\lambda_2 = M$ must use the positive sign and with a bit more algebra, we find that

$$\lambda_2 = M = m_N^R + m_N^L. \tag{78}$$

With these eigenvalues we find the eigenvectors, in terms of the right- and left-handed fields, to be [38]

$$N = (\nu_R + \nu_R^c) + \frac{m_D}{m_N^R}(\nu_L + \nu_L^c) \tag{79}$$

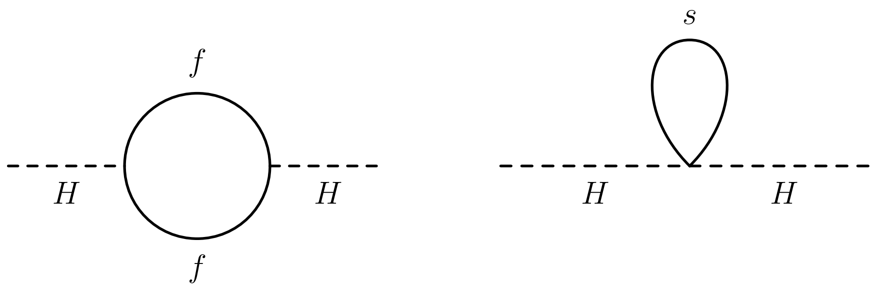
$$\nu = (\nu_L + \nu_L^c) - \frac{m_D}{m_N^R}(\nu_R + \nu_R^c) \tag{80}$$

Referencing our original assumption that $m_N \gg m_D$, we can see that N, the heavy neutrino, is essentially composed of only ν_R, while our light neutrino ν is almost only ν_L [38]. Similarily, we can say that right-handed neutrinos are composed of N and left-handed neutrinos are composed of ν [38]. This conclusion reflects what we observe: the neutrinos we detect are all left-handed and light, and we have not seen any heavier neutrinos because they are right-handed and do not interact via any of the Standard Model forces. The seesaw mechanism, then, can effectively explain why we observe such small neutrino masses.

3.2 Motivation for Supersymmetry

In addition to neutrino mass, additional clues indicate that we must look beyond the Standard Model for a more accurate description of Nature. Notably, the Standard Model does not describe gravity, whose quantum effects begin to be important at the reduced Planck scale $M_P = 1/\sqrt{8\pi G} = 2.4 \times 10^{18}$ GeV [42]. Contrasted with the electroweak scale at $M_W \sim 0.1$ to 1 TeV, we find the so-called hierarchy problem: why is gravity at a scale ten million billion times greater than the rest of the Standard Model [25]? This problem is not only inconvenient from an aesthetic perspective but also causes issues with the Higgs boson. We know from recent results that the Higgs boson has a mass of 125 GeV, but according to quantum field theory must receive quantum corrections from virtual effects of the particles that couple with it [42]. For example, the Higgs boson couples to the fermion as in Equation 28, so the schematic Feynman diagram in Figure 3a gives a correction to the original m_H:

$$\Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2}\Lambda_{UV}^2 + \dots, \tag{81}$$



(a) Dirac fermion loop contribution. (b) Scalar loop contribution.

Figure 3: One-loop contributions to the Higgs mass parameter [42].

where λ_f is the coefficient in the Higgs-fermion interaction term [42]. The ultraviolet momentum cutoff term Λ_{UV} regulates the loop integral and is the limit at which new, high-energy physics must enter the theory. However, when Λ_{UV} is close to the reduced Planck scale M_P, as we may expect, the correction becomes about 30 times greater than the value of m_H [42]. In effect, if the fermion is virtual it may take any mass up to this Planck scale mass, resulting in a Higgs boson with a mass near the Planck scale [53]. Of course, we have observed the Higgs boson's mass to be much nearer the electroweak scale than the Planck scale, so we must find an explanation for this fact.

The same problem arises even if we consider the particle in the loop correction to be a heavy scalar instead of a Dirac fermion. The Feynman diagram for this interaction is in Figure 3b and gives a correction to m_H^2:

$$\Delta m_H^2 = \frac{\lambda_s}{16\pi^2}\left(\Lambda_{UV}^2 + 2m_s^2 \ln(\Lambda_{UV}/m_s) + \ldots\right), \tag{82}$$

where m_s is the mass of the scalar [42]. We can use dimensional regularization to eliminate the Λ_{UV}^2 piece, but even then the correction will be sensitive to the mass of the scalar [42]. Since the scalar mass can be arbitrarily large, m_H can easily be higher than the electroweak scale.

How can we then explain the unavoidable fact that m_H is observed near the electroweak, not Planck, scale? The relative minus sign between Equations 81 and 82 suggests a symmetry between the Dirac and scalar particles: if each fermion is paired with two complex scalars where $\lambda_s = |\lambda_f|^2$, then the Λ_{UV}^2 terms cancel [42]. This convenient cancellation persists to all loop orders once we assume supersymmetry, which can be simply defined as:

$$Q|\text{boson}\rangle = |\text{fermion}\rangle \tag{83}$$

$$Q|\text{fermion}\rangle = |\text{boson}\rangle, \tag{84}$$

where the operator Q, along with $Q^\dagger$, is fermionic.

Though the intitial definition of supersymmetry is quite general, we can exactly specify the algebra. Haag, Sohnius and Lopuszanski proved that the following supersymmetry algebra is the only symmetry of the S-matrix that is consistent with relativistic quantum field theory:

$$\begin{aligned}
\{Q^A_\alpha, \bar{Q}_{\dot{\beta}B}\} &= 2\sigma^m_{\alpha\dot{\beta}} P_m \delta^A_B \\
\{Q^A_\alpha, Q^B_\beta\} &= \{\bar{Q}_{\dot{\alpha}A}, \bar{Q}_{\dot{\beta}B}\} = 0 \\
[P_m, Q^A_\alpha] &= [P_m, \bar{Q}_{\dot{\alpha}A}] = 0 \\
[P_m, P_n] &= 0,
\end{aligned} \tag{85}$$

where the Greek indices denote Weyl spinors and the Latin indices denote Lorentz four-vectors [57]. The capital indices denote the dimension of the supersymmetry; we shall be considering $N = 1$ unless otherwise noted.

One of the first conclusions in supersymmetry is that every representation of the algebra has an equal number of bosonic and fermionic states [57]. As a result, each particle in the Standard Model has a supersymmetric partner; this partner is bosonic for Standard Model fermions and fermionic for Standard Model bosons. These supersymmetric particles, if they are truly symmetric, should have the same mass as their Standard Model partner, but we have not yet observed any such particles at the appropriate mass scale. Therefore, supersymmetry is a broken symmetry, but we must be careful. Since unbroken supersymmetry solves the hierarchy problem so neatly, we must preserve the condition that $\lambda_s = |\lambda_f|^2$. For breaking the symmetry, then, we consider soft supersymmetry breaking [42]. "Soft" terms have positive mass dimension so that no divergences occur, and can arise from various sources, including supergravity [1]. The Lagrangian becomes

$$\mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{soft}, \tag{86}$$

where $\mathcal{L}_{SUSY}$ is the unbroken supersymmetry Lagrangian that we will discuss in the next section and $\mathcal{L}_{soft}$ violates supersymmetry [42]. As a result, the contribution to the Higgs mass squared term is

$$\Delta m^2_H = m^2_{soft}\left(\frac{\lambda}{16\pi^2}\ln(\Lambda_{UV}/m_{soft}) + ...\right), \tag{87}$$

where m_{soft} term determines the difference in mass between particles and their supersymmetric partners [42]. Because we want a small contribution to m_H we also require m_{soft} to be relatively small as well; the lightest supersymmetric particles are estimated to be on the TeV scale [42]. This scale is larger than the Standard Model particles, so the seesaw mechanism fits naturally into supersymmetry. Therefore, supersymmetry can easily incorporate the theories previously discussed involving massive and sterile neutrinos.

3.3 The Minimal Supersymmetric Standard Model

3.3.1 Particle Content

Before we discuss more complex supersymmetric models, we will review the Minimal Supersymmetric Standard Model (MSSM), which contributes the $\mathcal{L}_{SUSY}$ term in Equation 86 and forms the basis of the theories we will examine later.

First we will discuss the particle content of the MSSM: supersymmetry requires a bosonic partner for each Standard Model fermion and vice versa, so we need to define these new particles, called sparticles. Could these sparticles be a part of the Standard Model already? Note that supersymmetry does not affect the $SU(3)_C$, $SU(2)_L$ or $U(1)_Y$ degrees of freedom, so spin-half singlets have spin-0 singlet partners, etc. [1]. Take the lepton $SU(2)_L$ doublet–

$$L_m(x) = \begin{pmatrix} \nu_m \\ e_{Lm} \end{pmatrix}, \tag{88}$$

where $e_{Lm} = \mathcal{E}_m$ is the left-handed electron field, as an example. There is a potential partner doublet: the spin-0 $SU(2)_L$ Higgs doublet in Equation 10. However, the Higgs doublet does not carry lepton number, which is conserved, and so we need a new doublet that does satisfy this requirement:

$$\tilde{L}_m(x) = \begin{pmatrix} \tilde{\nu}_m \\ \tilde{e}_{Lm} \end{pmatrix}, \tag{89}$$

where $\tilde{\nu}$ is the neutrino scalar partner and $\tilde{e}_L$ is the electron scalar partner [1]. The right-handed lepton field e_{Rm} is in an $SU(2)_L$ singlet, so its supersymmetric partner $\tilde{e}_R$ is also a singlet. Likewise, quarks are triplets of $SU(3)_C$ and so we need a new scalar triplet. Since we have not been able to pair any of the fermions with Standard Model spin-one bosons, we also need new fermionic partners for the Standard Model bosons. However, while in the Standard Model the Higgs doublet simply has a charge conjugate, in the supersymmetric version we require

Particle	spin-0	spin-1/2	Gauge Transformation
Q	$\left(\tilde{u}_L\ \tilde{d}_L\right)$	$(u_L\ d_L)$	$\left(\mathbf{3}, \mathbf{2}, +\frac{1}{6}\right)$
u	$\tilde{u}_R$	u_R	$\left(\mathbf{3}, \mathbf{1}, +\frac{2}{3}\right)$
d	$\tilde{d}_R$	d_R	$\left(\mathbf{3}, \mathbf{1}, -\frac{1}{3}\right)$
L	$(\tilde{\nu}\ \tilde{e}_L)$	$(\nu\ e_L)$	$\left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}\right)$
e	$\tilde{e}_R$	e_R	$(\mathbf{1}, \mathbf{1}, -1)$
H_u	$(H_u^+\ H_u^0)$	$\left(\tilde{H}_u^+\ \tilde{H}_u^0\right)$	$\left(\mathbf{1}, \mathbf{2}, +\frac{1}{2}\right)$
H_d	$\left(H_d^0\ H_d^-\right)$	$\left(\tilde{H}_d^0\ \tilde{H}_d^-\right)$	$\left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}\right)$

Table 4: Chiral supermultiplets of the MSSM.

two separate doublets:

$$\begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} \text{ has the supersymmetric partner } \begin{pmatrix} \tilde{H}_u^+ \\ \tilde{H}_u^0 \end{pmatrix}$$
$$\begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} \text{ has the supersymmetric partner } \begin{pmatrix} \tilde{H}_d^0 \\ \tilde{H}_d^- \end{pmatrix}$$

for the two Higgs doublets H_u and H_d [1]. A summary of the particles in the MSSM (excluding the gauge supermultiplets) is found in Table 4.

3.3.2 The MSSM Lagrangian

With the particle content of the MSSM established, we now construct the Lagrangian for this theory.[12] We start with the Wess-Zumino model, which describes the kinetics of a massless complex scalar ϕ and a massless complex spinor ψ [42]:

$$\mathcal{L}_{free} = -\partial_\mu \phi^\dagger \partial^\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi. \tag{90}$$

These terms by themselves are not surprising, but ϕ has 2 degrees of freedom as it is one complex field while ψ has four as the spinor has two complex components [1]. In order to have the same number of degrees of freedom for both the scalars and fermions, we add an additional, non-interacting scalar field F:

$$\mathcal{L}_{free} = -\partial_\mu \phi^\dagger \partial^\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi + F^\dagger F. \tag{91}$$

[12] We will be ignoring any $D > 4$ operators [4].

To add interactions, we introduce terms involving functions of the bosonic fields [1]:

$$\mathcal{L}_{int} = W^i F_i - \frac{1}{2}\left(W^{ij}\psi_i\psi_j\right) + \text{h.c.}. \tag{92}$$

The function W is the superpotential

$$W = \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{1}{6}Y^{ijk}\phi_i\phi_j\phi_k \tag{93}$$

while W^i and W^{ij} are the first and second derivatives of W with respect to ϕ [1]. The symmetric in i, j M^{ij} is a mass matrix for the fermions and the symmetric in i, j, k Y^{ijk} is the Yukawa coupling between two fermions and a boson [42]. By eliminating the auxiliary scalar fields using their equations of motion, we find that $F_i = -W_i^\dagger$ and $F^{\dagger i} = -W^i$ [42]. Therefore, we have the complete Lagrangian for the chiral part of the theory:

$$\mathcal{L}_{chiral} = -\partial_\mu\phi^\dagger\partial^\mu\phi + i\psi^\dagger\bar{\sigma}^\mu\partial_\mu\psi - \frac{1}{2}\left(W^{ij}\psi_i\psi_j + \text{h.c.}\right). \tag{94}$$

Now we consider the gauge portion of the Lagrangian. As with the chiral supermultiplets, we must add an auxiliary field D. The Lagrangian is

$$\mathcal{L}_{gauge} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\lambda^\dagger\bar{\sigma}^\mu\partial_\mu\lambda + \frac{1}{2}D^2 \tag{95}$$

where the first two terms are the usual Maxwell term for the photon and a massless spinor term for the photino [1].

To combine these chiral and gauge supermultiplet terms into a single Lagrangian, we simply add Equations 91 and 95, but replace the derivatives in the chiral Lagrangian with the covariant derivative $D_\mu = \partial_\mu + iqA_\mu$ to make it gauge-invariant, where q is the $U(1)$ charge [1]. As a result, we get the Lagrangian

$$\mathcal{L}_{abelian} = -D_\mu\phi^\dagger D^\mu\phi + i\psi^\dagger\bar{\sigma}^\mu\partial_\mu\psi + F^\dagger F - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\lambda^\dagger\bar{\sigma}^\mu\partial_\mu\lambda + \frac{1}{2}D^2. \tag{96}$$

We do not use the purely chiral interaction terms that we found in Equation 92, but instead add interaction terms involving both gauge and chiral fields, which take the form

$$\mathcal{L}_{int} = -\sqrt{2}q\left[(\phi^\dagger\psi)\lambda + \lambda^\dagger(\psi^\dagger\phi)\right] - q(\phi^\dagger\phi)D; \tag{97}$$

the first two terms can be considered the supersymmetrification of the usual gauge/matter field coupling [1]. Using the same method as before, we eliminate the auxiliary field D so that the Lagrangian for the abelian case becomes

$$\begin{aligned}\mathcal{L}_{abelian} =& - D_\mu \phi^\dagger D^\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi + F^\dagger F - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\lambda^\dagger \bar{\sigma}^\mu \partial_\mu \lambda \\ & - \sqrt{2} q \left(\phi^\dagger \psi \lambda + \lambda^\dagger \psi^\dagger \phi\right) - \frac{1}{2} q^2 \left(\phi^\dagger \phi\right)^2 . \end{aligned} \tag{98}$$

For the non-abelian case, we must modify the gauge and interaction terms in the Lagrangian. The gauge terms become

$$\mathcal{L}_{gauge} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i\lambda^{\dagger a} \bar{\sigma}^\mu \nabla_\mu \lambda^a + \frac{1}{2} D^a D^a \tag{99}$$

where the first term now contains the Yang-Mills field strength and $\nabla_\mu \lambda^a$ is the covariant derivative for the photino [42]. The index a runs over the corresponding gauge group as in Table 1 and g is the appropriate gauge coupling constant. The interaction terms then become

$$\mathcal{L}_{int} = -\sqrt{2} g \left[(\phi^\dagger T^a \psi) \lambda^a + \lambda^{\dagger a} (\psi^\dagger T^a \phi) \right] - g(\phi^\dagger T^a \phi) D^a \tag{100}$$

where T^a is the generators for the gauge group [42]. We can use the same methods as before to eliminate the auxiliary fields, so the final Lagrangian is

$$\begin{aligned}\mathcal{L}_{SUSY} =& - D_\mu \phi^\dagger D^\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i\lambda^{\dagger a} \bar{\sigma}^\mu \nabla_\mu \lambda^a \\ & - \sqrt{2} g \left[(\phi^\dagger T^a \psi) \lambda^a + \lambda^{\dagger a} (\psi^\dagger T^a \phi) \right] \\ & - |W_i|^2 - \frac{1}{2} g^2 \left(\phi^\dagger T^a \phi\right)^2 . \end{aligned} \tag{101}$$

The first line defines the kinetics of the fields, the second line defines the interactions, and the last line of this equation is the scalar potential.[13]

To fully specify the MSSM, we must define the superpotential W of the theory:

$$W = Y_u^{ij} \bar{u}_i Q_j H_u - Y_d^{ij} \bar{d}_i Q_j H_d - Y_e^{ij} \bar{e}_i L_j H_d + \mu H_u H_d \tag{102}$$

where the 3×3 matrices Y^{ij} are the same Yukawa couplings as in the Standard Model [1]. Unlike the Standard Model, of course, these couplings also describe the interactions between the new supersymmetric particles as well. Therefore, μ is the only new parameter added by supersymmetry. Expanding out the last term

[13]Note that we have been using the component fields, not superfields, for clarity and because it is easier to compare to the Standard Model terms [42].

in Equation 102, we get a term for the Higgsino mass,

$$\mu\left(\bar{H}_u^+\bar{H}_d^- - \bar{H}_u^0\bar{H}_d^0\right) + \text{c.c.}, \tag{103}$$

and a supersymmetric Higgs mass-squared term,

$$|\mu|^2\left(|H_u^0|^2 + |H_u^+|^2 + |H_d^0|^2 + |H_d^-|^2\right); \tag{104}$$

this term appears in the scalar potential [42].

3.3.3 Problems with the MSSM

The MSSM has several problems. As with all theories, we want to avoid fine-tuning: large cancellations that are not the consequence of some correlation or symmetry in the UV theory [51]. However, in the MSSM the new parameter μ, which conserves supersymmetry, is correlated with the soft supersymmetry-breaking parameter; why is this the case [51]? In addition, μ must be on the order of 10^2 or 10^3 GeV, which is much lower than expected, so that the Higgs has the correct mass [42]. These issues are known as the μ problem. One possible solution is to replace the μ term by a Yukawa coupling involving H_u, H_d and a new scalar field with an appropriately-scaled vacuum expectation value [22]. This theory is known as the next-to-minimal supersymmetric standard model (NMSSM), and it comes in both R-parity conserving and violating varieties [14]. However, we will continue to consider the MSSM as R-parity violation may offer a solution to the μ problem.

In addition, supersymmetry must be a broken theory, but the Lagrangian in Equation 101 contains no supersymmetry-breaking terms. For the reasons we discussed in Section 3.2, we add $\mathcal{L}_{soft}$ terms. Some viable soft supersymmetry breaking terms are

$$\mathcal{L}_{soft} = -\left(\frac{1}{2}M_a\lambda^a\lambda^a + \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j\right) + \text{c.c.} - \left(m^2\right)^i_j\phi^{*j}\phi_i, \tag{105}$$

where c.c. represents the charge conjugated terms and M_a is the gaugino mass, b^{ij} and m^2 are scalar mass-squared terms, and a^{ijk} is a scalar cubic coupling term [42]. These terms clearly break supersymmetry because they only involve gauginos and scalars, giving them mass even if their Standard Model partners, the gauge bosons and fermions, have low or zero mass [42]. The seesaw mechanism could easily be applied in this situation. Note, though, that these terms should be generated by some as yet unknown spontaneous supersymmetry-breaking mechanism, and so currently only serve as "parameterisations of our ignorance" of this mechanism [6].

Lastly, and most importantly for this paper, the MSSM allows for baryon and lepton number violating interactions [7]. Recall that we introduced lepton and baryon number L and B in Equations 2 and 3. These quantities were conserved in the Standard Model, but things become more complicated after the introduction of supersymmetry. We attribute B and L to the spin-0 fermion superpartners, the squarks and sleptons, but have no insurance that these new baryon and lepton numbers are conserved [6]. In addition, with all these new bosons we also have to consider their potential exchanges with the fermions: if allowed, these interactions would lead to a dimension-four proton decay [7]. While Standard Model bosons are spin-one, the squarks and sleptons are spin-0, which suggests a way to prevent unwanted interactions [6]. This method will turn out to be R-parity, a generalization of B and L for supersymmetric theories.

4 R-Parity and its Violation

The primary motivation for the introduction of R-parity is to resolve the issue of baryon and lepton number violation. R-parity is associated with the discrete Z_2 subgroup of the continuous $U(1)_R$ R-symmetry transformations, and serves to forbid unwanted interactions between the sfermions and their Standard Model partners [6].

4.1 Why R-Parity?

The first question to resolve is why we use R-parity instead of the continuous version R-symmetry, or R-invariance. R-invariance is a symmetry of the MSSM Lagrangian where the operator R acts on fields as

$$\begin{aligned} R \cdot S &= P \\ R \cdot P &= -S \\ R \cdot \psi &= \frac{1}{2}\gamma_5 \psi; \end{aligned} \tag{106}$$

S is a scalar field, P is a pseudoscalar field, and ψ is a spinor field [26]. These equations give the first hint that R-invariance may not be the final answer: symmetries like R-symmetry that act via γ_5 are often quantum mechanical anomalies and so will be broken by quantum effects [26]. The main reasons for abandoning R-invariance come from the masses of the gravitino and gluinos: if the continuous $U(1)$ R-symmetry is not broken, these particles will not have mass [6]. As a result of the gravitino's masslessness, supersymmetry with added gravitation will not be spontaneously broken. Even with spontaneous supersymmetry breaking, R-invariance acts chirally on gluinos and renders them massless; however, massless gluinos would lead to the production of "R-hadrons," made from quarks, antiquarks and gluinos, that have simply not been observed [6]. For these theoretical and phenomenological reasons, we abandon the continuous R-invariance in favor of its discrete subgroup R-parity.

How do we define R-parity, then? R-transformations, as in Equation 106, are defined to not act on Standard Model particles, so we assign them $R = 0$ while superparticles have $R = \pm 1$ [6]. R-parity R_p is defined as

$$R_p = (-1)^R = \begin{cases} +1 \text{ for Standard Model particles} \\ -1 \text{ for supersymmetric particles} \end{cases} \tag{107}$$

where the particles with $R_p = +1$ are called R-even and particles with $R_p = -1$ are called R-odd [6].

R-parity offers a good way to differentiate between Standard Model and supersymmetric particles, but how does it prevent unwanted interactions? Several interactions in the MSSM superpotential violate lepton and baryon number:

$$W \supset \epsilon_i LiH_u + \lambda_{ijk} L_i L_j \bar{e}_k + \lambda'_{ijk} Q_i L_j \bar{d}_k + \lambda''_{ijk} \bar{u}_i \bar{d}_j \bar{d}_k; \tag{108}$$

the first three terms violate lepton number and the last term violates baryon number [1,7]. In addition to number violations, combining the last two terms produces the dimension-four contributions to proton decay, and if we assume $\lambda', \lambda'' \sim 1$ and $m_{squark} \sim$ TeV, then we get an unacceptably small proton lifetime: $\tau_P \sim 10^{-20}$ years instead of the experimentally observed $\tau_p > 10^{32-34}$ years [48]. Recalling that the main goal of R-parity is to prevent a combination of baryon and lepton number violation, we redefine R_p in terms of spin S, B and L:

$$R_p = (-1)^{2S} (-1)^{3B+L}; \tag{109}$$

the second term is called matter-parity symmetry [6]. By itself, the imposition of matter-parity forbids all unwanted interactions, but this solution is "overkill" as we do not need to forbid all the couplings in order to prevent fast proton decay [48]. Therefore, we will only be considering R-parity.

The definition of R-parity in Equation 109 can be rewritten as

$$R_p = (-1)^{2S} (-1)^{3(B-L)} \tag{110}$$

which shows that it is actually the quantity $B - L$ that is conserved by R-parity[14] [6]. Either baryon or lepton number may be violated, but not both at the same time.

[14]This connection was first shown by Weinberg and Farrar.

R-parity conservation has several beneficial consequences in the MSSM. First, as we have discussed, it prevents a combination of baryon and lepton number violation and thus the unphysical rapid decay of the proton. In addition, by preventing the exchange between Standard Model and supersymmetric particles, R-parity explains why we have not seen these types of interactions by ensuring that the supersymmetric particles can only be pair-produced (or at least produced in even numbers) [6, 42]. By the same reasoning, sparticles can only decay into other sparticles, resulting in a state with an odd number of the lightest supersymmetric particle, or LSP [42]. The LSP, which could be a neutralino, sneutrino or gravitino, then must be completely stable and, if it is electrically neutral and therefore only weaky interacting, is a candidate for dark matter [6, 42]. The advantages of including R-parity in the MSSM are clear.

4.2 Why R-Parity Violation?

Though the imposition of R-parity has convenient results, the symmetry itself is not physically motivated. Other symmetries that are discrete, continuous, global, or local can offer the same restrictions on proton decay as R-parity, but allow R-parity violating ($\not{R}_p$) terms [6]. In addition, the collider signatures of the MSSM depend on whether R-parity is conserved or violated [48]. Therefore, it is worthwhile to explore theories that allow for R-parity violation.

Either the superpotential W or soft supersymmetry breaking terms can give rise to $\not{R}_p$ terms [6]. We have already seen the $\not{R}_p$ couplings originating in the superpotential in Equation 108:

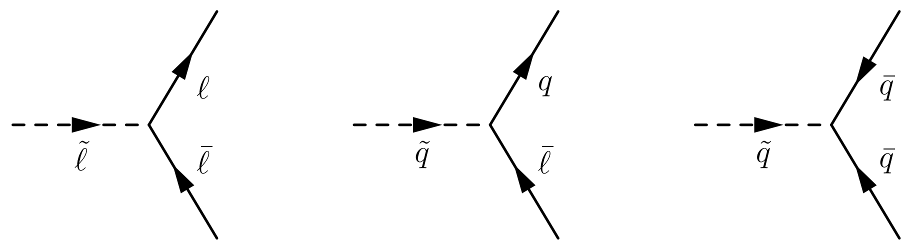
$$W_{\not{R}_p} = \epsilon_i L_i H_u + \frac{1}{2}\lambda_{ijk} L_i L_j \bar{e}_k + \lambda'_{ijk} Q_i L_j \bar{d}_k + \frac{1}{2}\lambda''_{ijk} \bar{u}_i \bar{d}_j \bar{d}_k. \tag{111}$$

The first term, $\epsilon_i L_i H_u$, is bilinear while the others are trilinear; the trilinear interactions are pictured in Figure 4. The $\not{R}_p$ superpotential nearly doubles the number of couplings in the MSSM and has 48 parameters from these $\not{R}_p$ couplings: 3 from ϵ_i, 9 from λ_{ijk} since it is antisymmetric in i and j, 27 from λ'_{ijk}, and 9 from λ''_{ijk} since it is antisymmetric in j and k [2, 6].

The soft symmetry-breaking terms that we discussed in Section 3.3.3 must also be expanded to incorporate $\not{R}_p$ terms. These new soft terms are

$$\begin{aligned} \mathcal{L}^{\not{R}_p}_{soft} =& \frac{1}{2} A_{ijk} \tilde{L}_i \tilde{L}_j \tilde{\bar{\ell}}_k + A'_{ijk} \tilde{L}_i \tilde{Q}_j \tilde{\bar{d}}_k + \frac{1}{2} A''_{ijk} \tilde{\bar{u}}_i \tilde{\bar{d}}_j \tilde{\bar{d}}_k \\ &+ B_i H_u \tilde{L}_i + \tilde{m}^2_{di} H_d^\dagger \tilde{L}_i + \text{h.c.}, \end{aligned} \tag{112}$$

where the A coupling constants have the same antisymmetry properties as the λ couplings in the superpotential, B is the bilinear coupling term, and $\tilde{m}^2_d$ is a soft



(a) The λ_{ijk} interaction. (b) The λ'_{ijk} interaction. (c) The λ''_{ijk} interaction.

Figure 4: The trilinear $\not{R}_p$ couplings [6]. Figures 4a and 4b violate lepton number and Figure 4c violates baryon number.

mass parameter [6]. All together, these terms add 51 new $\not{R}_p$ parameters.

With all these new parameters, we wish to impose constraints on their values. In theory, the couplings can induce very large baryon and lepton number violating effects since they are not suppressed by any large mass scale, but in reality our non-observations of such effects limit some of the couplings to be at very small scales (around 10^{-26}) [6]. In addition, further theoretical, astrophysical and cosmological bounds are imposed. We mention some constraints from recent LHC results in the last section.

4.3 Explicit R-Parity Violation

So far we have described how R-parity is violated, but not why. R-parity can be violated either through explicit additions to the MSSM or other theories, or may be generated spontaneously. With explicit R-parity breaking we can include the bilinear or trilinear terms from the superpotential or soft-symmetry breaking terms, but no matter the combination of terms, we must ensure that they allow for a consistent quantum field theory. A variety of combinations of terms are valid, but we shall focus on the two most popular scenarios:

- *R-parity breaking by bilinear terms:* in this scenario only the bilinear couplings ϵ_i, B_i, and $\tilde{m}^2_{di}$ are considered, for a total of 9 parameters. It is important to note, though, that without R-parity and lepton number conservation, no distinction exists between the $Y = -1$ Higgs field, H_d, and the lepton field L_m, as their gauge charges are the same [6]. Therefore, there is a choice in the weak interaction basis which leads to different values of lepton number violating couplings. For example, we can choose a basis in which $\epsilon_i = 0$ but non-zero slepton v.e.v.s are introduced by the soft bilinear $\not{R}_p$ terms, or choose a basis where these v.e.v.s remain zero but $\epsilon_i \neq 0$ [6]. In this bilinear-only theory, then, the λ and λ' trilinear terms are generated by rotating the weak eigenstate basis to the mass eigenstate basis, but as a result their parameters are not independent [6]. The advantage of this theory is its predictivity, and the main challenge lies in restricting the neutrino masses generated by the bilinear terms to an appropriate scale [6].

- *R-parity breaking by trilinear terms:* in this scenario only the trilinear λ couplings in the superpotential and their corresponding A couplings in the soft-symmetry breaking terms are included, for a total of 45 parameters. The advantage of this theory is that the ambiguity of the weak interaction basis is absent. However, note that the bilinear terms are generated by the soft-symmetry breaking terms as, in their presence, the bilinear terms in the superpotential and scalar potential cannot be simultaneously rotated away [6]. However, because experimental results limit the allowed amount of bilinear $\not{R}_p$ we can consider trilinear-only theories without much loss of accuracy [6].

Most theories incorporate both types of $\not{R}_p$ terms, and we will discuss their effects on neutrino masses later.

4.4 Spontaneous R-Parity Violation

Instead of manually inserting terms into the MSSM, we can also allow R-parity to be violated spontaneously. The simplest way to accomplish this violation is to give a vacuum expectation value to a sneutrino [6]. Predictably, a Goldston boson is generated by the spontaneous lepton number breaking, called the majoron [7]. The majoron and its scalar partner ρ form a decay mode for the Z-boson that is not observed in experiments [6]. Several solutions exist to this problem: (1) introduce some explicit lepton number violating terms into the MSSM in order to give the majoron mass, (2) expand the gauge group to include lepton number so that the majoron becomes the longitudinal component of a new gauge boson, or (3) use a non-zero v.e.v. of the new right-handed sneutrino so that the majoron is an electroweak singlet and so cannot contribute much to the Z-boson decay [6].

4.4.1 Mechanism for Spontaneous $\not{R}_p$

The simplest solution both the majoron mass and decay problems is (2), where we extend the MSSM algebra to include $B - L$ symmetry [48]:

$$G_{B-L} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}. \tag{113}$$

Each of the superfields in the MSSM gains a new quantum number, as noted in Table 5. In addition, we must introduce chiral superfields for the right-handed neutrinos $\hat{\nu}^c$ in order to cancel anomalies [7]. This new $U(1)_{B-L}$ symmetry must

Field	$\mathbf{SU(3)_C}$	$\mathbf{SU(2)_L}$	$\mathbf{U(1)_Y}$	$\mathbf{U(1)_{B-L}}$
$\hat{Q} = (\hat{u}, \hat{d})$	3	2	$+1/6$	$+1/3$
$\hat{u}^c$	$\bar{3}$	1	$-2/3$	$-1/3$
$\hat{d}^c$	$\bar{3}$	1	$-1/3$	$-1/3$
$\hat{L} = (\hat{\nu}, \hat{e})$	1	2	$-1/2$	-1
$\hat{e}^c$	1	1	-1	$+1$
$\hat{\nu}^c$	1	1	0	$+1$
$\hat{H}_u$	1	2	$+1/2$	0
$\hat{H}_d$	1	2	$-1/2$	0

Table 5: A list of the MSSM superfields and their quantum numbers [48].

be broken in order to get back to the SM/MSSM gauge symmetry. While breaking $B-L$ symmetry by an even charge conserves R-parity, breaking $B-L$ symmetry by an odd charge, as is the case with the sneutrino's charge of $+1$, results in R-parity violation [47]. Therefore, with a right-handed sneutrino v.e.v. both the $U(1)_{B-L}$ and R-parity symmetries are spontaneously broken and the majoron becomes the longitudinal component of the Z'-boson (the Z-boson associated with the broken $B-L$ symmetry), effectively resolving all of our problems [7].

R-parity *can* be conserved in this model if the v.e.v. of the majoron is non-zero while $\langle \tilde{\nu}^c \rangle = 0$ [47]. The conditions for minimizing the majoron $x, \bar{x}$ are

$$\frac{1}{2} M_{Z'}^2 = -|\mu_x|^2 + \frac{m_x^2 \tan^2 z - m_{\bar{x}}^2}{1 - \tan^2 z} \tag{114}$$

where $\tan z = x/\bar{x}$ and $M_{Z'} = g_{B-L}^2 (x^2 + \bar{x}^2)$ is the mass of the Z'-boson [47]. In the limit $x \gg \bar{x}$ with $m_x^2 < 0$ and $m_{\bar{x}}^2 > 0$, the minimization conditions become

$$\frac{1}{2} M_{Z'}^2 = -|\mu_x|^2 - m_x^2. \tag{115}$$

The left-hand side of this equation is positive, so we must have $-m_x^2 > |\mu_x|^2$ to have spontaneous $B-L$ violation [47]. This relationship is reminiscent of the relationship between μ and m_{H_u} in the MSSM: the μ problem [47]. Therefore, insisting that R-parity is conserved leads to an additional μ-like problem, which is clearly not ideal.

This problem can be avoided if R-parity is allowed to be violated. Evaluating the minimization conditions in Equation 114 in the limit where the $n \gg x, \bar{x}$ and

$g_{B-L}^2 \ll 1$, we find the v.e.v.s of the snuetrino and majoron:

$$n^2 = \frac{-m_{\tilde{\nu}^c}^2 \Lambda_{\bar{x}}^2}{f^2 m_{\bar{x}}^2 + \frac{1}{8} g_{B-L}^2 \Lambda_{\bar{x}}^2} \tag{116}$$

$$\bar{x} = \frac{-m_{\tilde{\nu}^c}^2 f \mu_x}{\sqrt{2}\left(f^2 m_{\bar{x}}^2 + \frac{1}{8} g_{B-L}^2 \Lambda_{\bar{x}}^2\right)} \tag{117}$$

$$x = \frac{-m_{\tilde{\nu}^c}^2 \left(a_x \Lambda_{\bar{x}}^2 + f b_x \mu_x\right)}{-m_{\tilde{\nu}^c}^2 \Lambda_{\bar{x}}^2 \left(2f^2 - \frac{1}{4} g_{B-L}^2\right) + f^2 m_{\bar{x}}^2 \Lambda_x^2 + \frac{1}{8} g_{B-L}^2 \Lambda_{\bar{x}}^2 \Lambda_x^2}, \tag{118}$$

where $\Lambda_x^2 = \mu_x^2 + m_x^2$ and $\Lambda_{\bar{x}}^2 = \mu_x^2 + m_{\bar{x}}^2$ [47]. For the v.e.v.s to be positive and $B - L$ symmetry to be broken, we require that $m_{\tilde{\nu}^c}^2 < 0$. Unlike the R-parity conserving case, the mass term is not linked to the μ parameter and so we do not get another μ problem.

We will find, in fact, that in most of parameter space R-parity is unavoidably, spontaneously broken. In the minimal $B - L$ model, the superpotential is

$$W_{B-L} = Y_u \hat{Q} \hat{H}_u \hat{u}^c + Y_d \hat{Q} \hat{H}_d \hat{d}^c + Y_e \hat{L} \hat{H}_d \hat{e}^c + Y_\nu \hat{L} \hat{H}_u \hat{\nu}^c + \mu \hat{H}_u \hat{H}_d \tag{119}$$

which we can use to find the v.e.v. of the right-handed sneutrino [48]:

$$\langle \tilde{\nu}^c \rangle = \sqrt{\frac{-4 m_{\tilde{\nu}^c}^2}{g_{B-L}^2}}. \tag{120}$$

As before, we see that the sneutrino mass is negative in order to have a consistent symmetry breaking mechanism [48]. For specificity, we implement the radiative symmetry breaking mechanism with the minimal supergravity boundary condition as in Ref. [47]. We choose this mechanism as it requires that one of the masses be negative, the role which the sneutrino fills. The next step is to identify the parts of the parameter space in this theory with R-parity violation and conservation. The parameters we will use are $f = \mathrm{diag}(f_1, f_2, f_3)$, which we can use to find the soft masses of the theory [48]. In Figure 5, where the red dots indicate spontanous R-parity violation and blue dots indicate R-parity conservation, we clearly see that R-parity violation dominates the parameter space [48]. The f parameters that allow for R-parity conservation are those that lead to degenerate right-handed neutrinos [48]. These results, though determined in a specific framework, can be reached in any $B - L$ symmetry extension of the MSSM [47]. Therefore, we conclude that in most $B - L$ theories, spontaneous R-parity violation will be required.

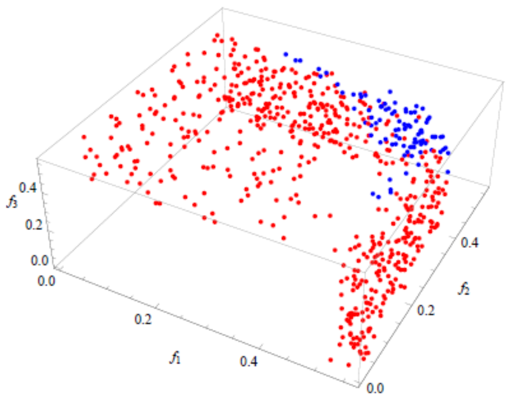


Figure 5: Plot of the $B - L$ breaking vacua in f-space, with all physical masses positive [47]. Red is R-parity violation and blue is R-parity conservation.

4.4.2 Minimal $U(1)_{B-L}$ Theory

Now that we have shown that R-parity violation occurs in minimal expansions of the MSSM, we shall examine this theory in more detail. We have already defined the superpotential in Equation 119, so we need to specify the soft terms [7]:

$$\begin{aligned}\mathcal{L}_{soft}^{B-L} =& m_{\tilde{\nu}^c}^2|\tilde{\nu}^c|^2 + m_{\tilde{L}}^2|\tilde{L}|^2 m_{\tilde{e}^c}^2|\tilde{e}^c|^2 + m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 \\ &+ \left(\frac{1}{2}m_{B-L}\tilde{B}'\tilde{B}' + A_\nu^D \tilde{L}^\top i\sigma_2 H_u \tilde{\nu}^c + B\mu H_u^\top i\sigma_2 H_d + \text{h.c.}\right) + \mathcal{L}_{soft}^{MSSM}.\end{aligned} \tag{121}$$

As designed, the v.e.v.s of the left- and right-handed sneutrinos, $\langle\tilde{\nu}\rangle = v_L/\sqrt{2}$ and $\langle\tilde{\nu}^c\rangle = v_R/\sqrt{2}$, generate the mass of the gauge bosons while also breaking R-parity and lepton number. We also find that

$$\langle\tilde{\nu}\rangle = \frac{B_\nu \langle\tilde{\nu}^c\rangle}{\sqrt{2}\left(m_{\tilde{L}} - \frac{1}{8}g_{B-L}^2\langle\tilde{\nu}^c\rangle^2\right)} \tag{122}$$

where $B_\nu = \frac{1}{\sqrt{2}}\left(Y_\nu \mu v_d - A_\nu^D v_u\right)$ [7].[15] The hierarchy between the left- and right-handed neutrinos is preserved as $\langle\tilde{\nu}^c\rangle \gg \langle\tilde{\nu}\rangle$.

With R-parity broken and lepton number violated, the quantum numbers that

[15] Y_ν is from the term added to the superpotential for the right-handed neutrinos $Y_\nu^{ij}\hat{L}_i\hat{\nu}_j^c\hat{H}_u$, and v_u and v_d are related to the v.e.v.s of the Higgs doublets: $\langle H_u\rangle = v_u/\sqrt{2}$ and $\langle H_d\rangle = v_d/\sqrt{2}$ [7].

distinguish between the leptons, Higgsinos and gauginos are eliminated and so mixing between these sectors occurs in the generated bilinear terms. Most of these $\not{R}_p$ bilinear terms are suppressed by neutrino masses, but the term $\frac{1}{2}g_{B-L}\nu^c\left(\nu^c\tilde{B}'\right)$ is not [7]. Trilinear $\not{R}_p$ terms are generated once neutralinos are integrated out of the theory, but the resulting interactions are negligible considering observed neutrino masses [7].

Lastly, we will describe the particle content and mass spectrum of this minimally $\not{R}_p$ theory, confirming that it is realistic. Most of the particles are the same as in the MSSM, but in the gauge boson sector has an additional neutral boson from the breaking of $B-L$, the Z'-boson. The main effect of R-parity breaking is allowing mixing between particle families, such as the neutralinos and neutrinos: $\nu, \nu^c, \tilde{B}, \tilde{B}', \tilde{W}_L^0, \tilde{H}_d^0$ and $\tilde{H}_u^0$ [7]. In the simple case where the left-handed neutrino v.e.v. goes to zero and the neutrino Yukawa term is small, we get the neutrino masses

$$\begin{aligned} M_\nu &= M_{seesaw} + M_{\not{R}_p} \\ &= \frac{1}{4}Y_\nu\left(M_{B-L} + \sqrt{4M_{Z'}^2 + M_{B-L}^2}\right)^{-1}\left(Y_\nu\right)^\top v_u^2 + mM_{\tilde{\chi}^0}^{-1}m^\top \end{aligned} \tag{123}$$

where $m = \mathrm{diag}(0,0,0,0,Y_\nu v_R/\sqrt{2})$ and $M_{\tilde{\chi}^0}$ is the MSSM neutralino mass matrix [7]. Mixing also occurs between the charged leptons and charginos, $e^c, \tilde{W}_L^+, \tilde{H}_u^+$ and $e, \tilde{W}_L^-, \tilde{H}_d^-$, but since the mixing is proportional to the left-handed neutrinos and neutrino Yukawa term, its contributions are small [7].

The last sector to analyze is the scalars. Recall that we must specify a weak interaction basis for the sleptons and Higgsinos. We choose the basis [7]

$$\begin{aligned} \sqrt{2}\mathrm{Im}(\tilde{\nu}, \tilde{\nu}^c, H_d^0, H_u^0) &\qquad \text{for the CP-odd scalars,} \\ \sqrt{2}\mathrm{Re}(\tilde{\nu}, \tilde{\nu}^c, H_d^0, H_u^0) &\qquad \text{for the CP-even scalars,} \\ (\tilde{e}^*, \tilde{e}^c, H_d^{-*}, H_u^+) &\qquad \text{for the charged scalars.} \end{aligned}$$

In the simple limit from before, the charged scalar mass matrices decouple into two eigenvalues representing the mass of the left- and right-handed sleptons and the MSSM mass matrix for the Higgs fields [7]. For the CP-odd sleptons, the first eigenvalue corresponds to the majoron "eaten" by the Z'-boson and is simply the imaginary part of the right-handed sneutrino, while the second eigenvalue is the mass of the (physical) left-handed sneutrino:

$$m_{\tilde{\nu}}^2 = M_{\tilde{L}}^2 - \frac{1}{8}g_{B-L}^2 v_R^2 - \frac{1}{8}\left(g_1^2 + g_2^2\right)\left(v_u^2 - v_d^2\right), \tag{124}$$

indicating that $M_{\tilde{L}}^2 > \frac{1}{8} g_{B-L}^2 v_R^2$ [7]. The mass $M_{\tilde{L}}$ is an eigenvalue for the CP-even sleptons, while the other CP-even eigenvalue is the mass of the real part of the right-handed sneutrino:

$$m_{\mathrm{Re}(\tilde{\nu}^c)}^2 = \frac{1}{4} g_{B-L}^2 v_R^2, \tag{125}$$

which is degenerate with the Z'-boson mass, $M_{Z'}^2 = \frac{1}{4} g_{B-L}^2 v_R^2$ [7]. In these cases, we find that the masses are realistic and equivalent to the MSSM mass modified by $B - L$ D-term contributions [7]. This result indicates that this minimal $U(1)_{B-L}$ theory is a possibility, but the observation or not of the theory's collider signatures will ultimately confirm or disprove it.

4.5 A Question of Naturalness

A concern that has arisen recently is the naturalness of supersymmetry, especially the MSSM. The idea of naturalness drove us to look for an explanation for the hierarchy problem, leading to the development of supersymmetry [25]. However, results from the LHC place bounds on the supersymmetric partners and their non-observation has led to tension between the desire for naturalness (that is, no fine-tuning) in the theory and the experimental results. Either we can continue to use the MSSM as our primary model and lessen the requirement of naturalness, allowing us to fine-tune particle masses to appropriate scales, or we can continue to insist on naturalness and abandon the MSSM in favor of theories that can more easily accomodate experimental results [54]. For example, the Higgs mass in the MSSM should be around 90 GeV, but has been observed to be 125 GeV [54]. We will approach R-parity violation from a naturalness perspective and mention other theories that satisfy the same requirements.

First, though, we must quantify what we mean by "naturalness." A rough definition is

$$\mathcal{N}^0 \equiv \frac{m_{H \text{ 1-loop}}^2}{m_H^2}, \tag{126}$$

where $m_H = 125$ GeV is the physical Higgs mass and $m_{H \text{ 1-loop}}$ is the one-loop correction to this mass, which is dependent on the cutoff scale Λ_{UV} and Yukawa Higgs-fermion coupling λ_f [25]. In the Standard Model with $\Lambda_{UV} \sim M_P$ and $\lambda_{top} \sim 1$, we find $\mathcal{N}^0 \sim 10^{30}$ [25]. As we found in Section 3.2, broken supersymmetry reduces the dependence of $m_{H \text{ 1-loop}}^2$ on Λ_{UV} from quadratic to logarithmic. Therefore, if the mass of the sfermion is not too much greater than the mass of the Higgs then $\mathcal{N}^0$ is only around 100, a much more natural result [25]. Of course, it is still up for debate exactly how much fine-tuning or how little naturalness is acceptable, but this method gives us a process by which we can evaluate theories.

One of the major tensions with the LHC results is the non-observation of

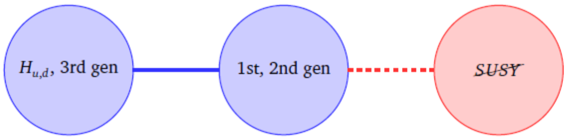


Figure 6: A sketch of natural, split supersymmetry [54].

the first and second generation superparticles, especially the squarks. A natural explanation for this is the idea of a split supersymmetry, where the Higgs and third generation, due to being strongly coupled, are insulated from supersymmetry breaking while the first and second generations experience it more strongly, as pictured in Figure 6 [54]. As a result, the third generation has soft masses less than the the masses of the first and second generation, explaining why we have not seen these heavier particles yet. Because of this splitting, the soft masses of the two generation groups have different sources [54]. This theory effectively creates an inverted hierarchy in the superpartners. In this theory, R-parity violation can allow all the generations to have degenerate masses as it hides their signatures in collider experiments [54].

The inverted hierarchy created by split supersymmetry also arises in theories designed to resolve the Higgs mass problem. One such theory is effective supersymmetry, where the inverted hierarchy is realized in various ways [25]. In this theory, extra fields may be added to the effective theory to raise the Higgs mass, and so the visible particles are the supersymmetric third generation and possibly these new particles [25]. On the other hand, a class of theories called focus point supersymmetry defines *all* generations to be heavy; this pattern comes from an approximate $U(1)_R$ symmetry [25]. In addition to the splitting between generations, there is a large mass difference between the gluinos and squarks and the lighter superparticles [25]. Problematically, this hierarchy would allow the heavier particles to decay into energetic particles, leading to distinct signals (that have not been observed), as well as decreased naturalness due to the bounds on the lighter particles [25]. However, another theory called compressed supersymmetry solves this by making the superpartner spectrum degenerate. A summary of these theories and their strengths can be found in Table 6.

These natural theories explain the lack of supersymmetry observations at the LHC, but R-parity can also fulfill that task. By allowing superparticles to decay, the particles are effectively hidden from detection [54]. Many possibilities for R-parity violation exist, but similarities are present: not all the couplings can be large, and strong bounds on some of the individual parameters [25]. Though the theory by itself lacks a few of the features in Table 6, when added to pre-existing

	Effective	Focus Point	Compressed	$R\!\!\!/_p$
Naturalness	✓	✓	✓	✓
Grand Unification	✓	✓	✓	✓
WIMP Dark Matter	✓	✓	✓	
LHC Null Results	✓	✓	✓	✓
Higgs Mass		✓		
Flavor/CP Constraints	✓	✓		

Table 6: A summary of the strengths and attributes of natural SUSY theories [25].

theories, such as the MSSM or split supersymmetry, R-parity violation can improve the naturalness of theories while helping to explain the null results at the LHC.

5 Neutrino Mass in R-Parity Violating Theories

In the Standard Model, implementing neutrino mass involves adding sterile neutrinos or modifying the Higgs sector, but in the MSSM, such measures are not necessary. If lepton number is violated,[16] as in R-parity violating versions of the MSSM, then neutrinos gain Majorana masses without any new fields. Therefore, neutrino mass is intrinsically supersymmetric [56]. As non-Standard Model fields have not been experimentally verified, $R\!\!\!/_p$ theories have a clear phenomenological advantage.

In addition to generating neutrino masses, any successful theory must also generate the correct mixing angles and hierarchy between the neutrinos. The current experimental values for the mass squared differences and mixing angles are

$$\begin{aligned} &\Delta m_{23}^2 = 2.0 \times 10^{-3}\ \text{eV}^2, \qquad \Delta m_{12}^2 = 7.2 \times 10^{-5}\ \text{eV}^2, \\ &\sin^2\theta_{23} = 0.5, \qquad \sin^2\theta_{12} = 0.3, \qquad \sin^2\theta_{13} < 0.074, \end{aligned} \tag{127}$$

so $\theta_{23} \sim 45°$ is maximal, $\theta_{12} \sim 30°$ is large, and $\theta_{13} \leq 15°$ is small [52]. Neutrino oscillation experiments do not impose a scale on neutrino mass, but we can see a hierarchy: the first and second neutrinos are close in mass while the third neutrino is much larger (or smaller). In this paper, we will assume $m_1 < m_2 \ll m_3$. Based the WMAP results, we can determine, if m_1 is negligible, $m_2 \sim 0.008$ eV and $m_3 \sim 0.04$ eV [52]. These bounds present a challenge, as $R\!\!\!/_p$ couplings may

[16]Note that if lepton number is violated, baryon number is not in order to ensure a suitably long proton lifetime.

generate neutrino masses that are too large by several orders of magnitude [6]. In addition, mass hierarchies are generally associated with small mixing angles, so we must construct theories carefully in order to ensure both large mixing angles and an appropriate mass hierarchy [30]. We will examine the general effects of various contributions to neutrino masses before describing various specific theories.

5.1 General Mass Contributions

R-parity violating theories generate neutrino mass in two types of ways: tree-level contributions from bilinear $\not{R}_p$ and loop-level contributions from both trilinear and bilinear $\not{R}_p$ terms. In general, the exact leading effects from these contributions are model-dependent, but we can describe their effects in broad terms. We will use the weak interaction basis where $v_m = 0$, $v_0 = v_d$, $\tan\beta \equiv v_u/v_d$ and the down-type quark and lepton mass matrices are [30]

$$(m_d)_{nm} = \frac{1}{\sqrt{2}} v_d \lambda'_{0nm}, \qquad (m_\ell)_{nm} = \frac{1}{\sqrt{2}} v_d \lambda_{0nm}. \tag{128}$$

Note that the physical observables are independent of basis, so the results we find here are general.

Each of the contributions that we will examine involves two $\not{R}_p$ parameters. These interactions can be trilinear with λ-type couplings; lepton/Higgsino mass insertions, with μ_i and v_i where v_i can also be left-right mixing mass insertion parameter; or soft mass insertions, with B_i and $\tilde{m}^2_{d_k}$ [6]. Since the generation of Majorana neutrino mass depends on lepton number L being broken by two units, and each $\not{R}_p$ interaction violates L by one unit, we must have two such terms in order for the masses to be generated.

5.1.1 Tree-Level ($\mu\mu$) Contributions

We begin with the tree-level contributions to neutrino mass, pictured in Figure 7, which arise from the $\not{R}_p$ bilinear terms. At the tree level in supersymmetric theories, the three neutrinos mix with the four neutralinos, so the tree level neutral fermion mass matrix is 7×7 [52]:

$$\begin{pmatrix} M_1 & 0 & m_Z \sin\theta_W \frac{v_u}{v} & -m_Z \sin\theta_W \frac{v_d}{v} & 0 \\ 0 & M_2 & -m_Z \cos\theta_W \frac{v_u}{v} & m_Z \cos\theta_W \frac{v_d}{v} & 0 \\ m_Z \sin\theta_W \frac{v_u}{v} & -m_Z \cos\theta_W \frac{v_u}{v} & 0 & \mu & \mu_i \\ -m_Z \sin\theta_W \frac{v_d}{v} & m_Z \cos\theta_W \frac{v_d}{v} & \mu & 0 & 0 \\ 0 & 0 & \mu_i & 0 & 0 \end{pmatrix}. \tag{129}$$

Figure 7: Diagram for tree-level mass insertion. The μ points represent a mixing between the neutrino and a higgsino, while the cross in the center represents a neutralino Majorana mass term [30].

This matrix is in the $\left\{\tilde{B}, \tilde{W}_3, \tilde{H}_u, \tilde{H}_d, \nu_i\right\}$ basis, where M_1 is the bino mass, M_2 is the wino mass, and μ_i is a parameter from the superpotential[17] [30]. The neutralinos can be integrated out to get the neutrino mass matrix,

$$[m_\nu]_{ij}^{\mu\mu} = X_{tree}\mu_i\mu_j, \tag{130}$$

where

$$X_{tree} = \frac{m_Z^2 m_{\tilde{\gamma}} \cos^2\beta}{\mu\left(m_Z^2 m_{\tilde{\gamma}} \sin 2\beta - M_1 M_2 \mu\right)}, \tag{131}$$

with $m_{\tilde{\gamma}} \equiv \cos^2\theta_W M_1 + \sin^2\theta_W M_2$ [30]. If we assume, as we will from now on, that the relevant masses are at the electroweak scale $\bar{m}$, then Equation 130 simplifies to

$$[m_\nu]_{ij}^{\mu\mu} \simeq \frac{\cos^2\beta}{\bar{m}}\mu_i\mu_j. \tag{132}$$

The contributions to the neutrino masses at tree level are the eigenvalues of the neutrino mass matrix, so we assign the only non-zero value to the heaviest neutrino, $m_3^{tree} = X_{tree}\left(\mu_1^2 + \mu_2^2 + \mu_3^2\right)$ [30].

We can also write the resultant neutrino mass in a basis-invariant way:

$$m_3^{tree} = X_{tree}\mu^2 \sin^2\xi, \tag{133}$$

where ξ measures the separation between ν_i and μ_i as $\cos\xi = \sum_i v_i\mu_i/v_d\mu$ [52]. The alignment of ν_i and μ_i creates an appropriately small neutrino mass, but in general the two values are not correlated. Therefore, some amount of fine-tuning is required unless we find a mechanism that generates their alignment [52]. In any case, the main point of this discussion is that, in general, we cannot avoid giving mass to one of the neutrinos through tree level contributions.

[17] Note that $|\mu|^2 = \sum_i |\mu_i|^2$ [52].

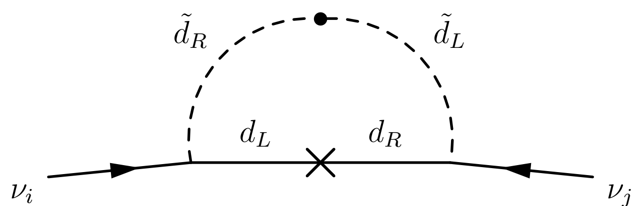


Figure 8: Diagram for a λ' loop contribution. The squark changes from right-handed to left-handed at the point and the cross represents a mass insertion [52].

5.1.2 Trilinear ($\lambda\lambda$ and $\lambda'\lambda'$) Loop Contributions

Neutrinos also receive mass from fermion-sfermion loops that depend on the trilinear terms: λ associated with the charged leptons and λ' associated with the down-type quarks. From the charged lepton loops, we get a contribution of

$$[m_\nu]_{ij}^{\lambda\lambda} \simeq \sum_{l,k} \frac{1}{8\pi^2} \lambda_{ilk}\lambda_{jkl} \frac{m_{\ell_l} \Delta m^2_{\tilde{\ell}_k}}{m^2_{\tilde{\ell}_k}}, \tag{134}$$

which is of the order

$$[m_\nu]_{ij}^{\lambda\lambda} \sim \sum_{l,k} \frac{1}{8\pi^2} \lambda_{ilk}\lambda_{jkl} \frac{m_{\ell_l} m_{\ell_k}}{\bar{m}} \tag{135}$$

in the electroweak mass scale approximation [52]. Likewise, the down-type quark loop contribution is

$$\begin{aligned} [m_\nu]_{ij}^{\lambda'\lambda'} &\simeq \sum_{l,k} \frac{3}{8\pi^2} \lambda'_{ilk}\lambda'_{jkl} \frac{m_{d_l} \Delta m^2_{\tilde{d}_k}}{m^2_{\tilde{d}_k}} \\ &\sim \sum_{l,k} \frac{3}{8\pi^2} \lambda'_{ilk}\lambda'_{jkl} \frac{m_{d_l} m_{d_k}}{\bar{m}}, \end{aligned} \tag{136}$$

where the 3 in the numerator arises because of the color factor for quarks [30]. Figure 8 shows this interaction. These equations are only approximations because we have ignored the quark flavor mixing.

These loop-generated masses are doubly Yukawa suppressed by the $\not{R}_p$ couplings λ (λ') and the charged lepton (down-type quark) mass in the denominator [30]. Since these fermion masses are relatively large, the trilinear contributions are not usually significant.

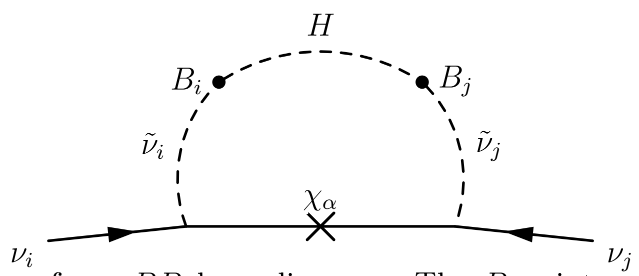


Figure 9: Diagram for a BB loop diagram. The B points represent a mixing between the sneutrinos and neutral Higgs bosons [30].

5.1.3 Bilinear Loop Contributions

If R-parity is violated by bilinear terms at the tree level, it is violated by bilinear terms at the loop level as well. Sneutrinos and neutral Higgs bosons mix at the tree level which also mixes the sneutrinos and antisneutrinos, leading to a splitting between the sneutrino eigenstates, which in turn generates Majorana neutrino mass at the loop level [52]. The B_i parameters are the couplings between the sneutrinos and Higgs, so we call these contributions BB loops, as shown in Figure 9. In the electroweak mass scale, the contributions to the neutrino mass matrix is

$$\begin{aligned} [m_\nu]_{ij}^{BB} &\sim \frac{g^2}{64\pi^2 \cos^2\beta} \frac{B_i B_j}{\bar{m}^3} \\ &= C_{ij} B_i B_j \end{aligned} \tag{137}$$

where C_{ij} is a matrix [30]. If the sneutrinos are degenerate, then C_{ij} is rank one and so only one neutrino will gain mass from this loop contribution [52]. Is this neutrino m_3 as in the case of the tree level bilinear contributions? Since in general B_i is not related to μ_i, we can say that m_2 gets mass from BB loops and assign m_1 to remain massless. If the sneutrinos are non-degenerate, then the mass of m_1 is a measure of their non-degeneracy [52]. Overall, contributions from BB loops are suppressed by the couplings B_i, which may be small, and $\cos^2\beta$. However, these suppressions are not as great as those on the other loop contributions, so BB loops are the dominant loop contribution [30].

The two types of bilinear terms can be combined to contribute to neutrino mass, resulting in μB loop contributions. These loops are also a result of the neutrino-Higgs and snuetrino-Higgs boson mixings [52]. In the electroweak mass scale, the contribution to the neutrino mass matrix is

$$[m_\nu]_{ij}^{\mu B} \sim \frac{g^2}{64\pi^2 \cos^2\beta} \frac{\mu_i B_j + \mu_j B_i}{\bar{m}^2}; \tag{138}$$

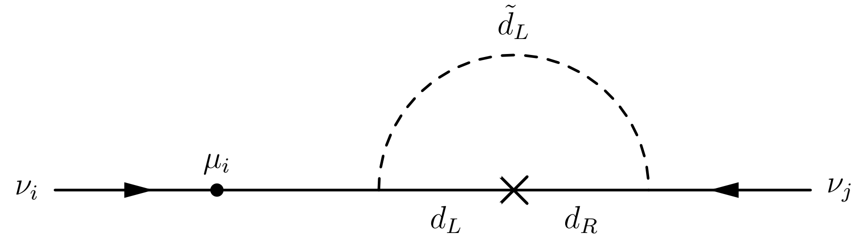


Figure 10: Diagram for a $\mu\lambda'$ loop diagram [52]. Note that the first part is from the tree-level diagram while the loop is from the λ' loop contribution diagram. The $\mu\lambda$ and μB diagrams can be constructed in a similar way.

since it depends on both the μ and B couplings, as well as $\cos^2\beta$, the effect of these loops is second-order if the tree level is dominant [30]. Therefore, these terms are not significant.

5.1.4 Mixed Loop Contributions

The combinations $\mu\lambda$ and $\mu\lambda'$ depend on both bilinear and trilinear $\not{R}_p$ and also contribute to the neutrino masses. Using the electroweak mass scale as usual, the contribution to the neutrino mass matrix from the $\mu\lambda$ loops is

$$[m_\nu]^{\mu\lambda}_{ij} \sim \sum_k \frac{1}{8\pi^2} g m_{\ell_k} \frac{\mu_i \lambda_{jkk} + \mu_j \lambda_{ikk}}{\bar{m}}. \tag{139}$$

The contribution from the $\mu\lambda'$ loops, shown in Figure 10, is similar,

$$[m_\nu]^{\mu\lambda'}_{ij} \sim \sum_k \frac{3}{8\pi^2} g m_{d_k} \frac{\mu_i \lambda'_{jkk} + \mu_j \lambda'_{ikk}}{\bar{m}}, \tag{140}$$

where we ignore squark mixing as before [30]. Compared to the $\lambda\lambda$ and $\lambda'\lambda'$ loops, these diagrams only have one Yukawa suppression instead of two; however, when the tree level contributions are dominant these loops become second order, as in the case of the μB loops [52]. Again, these contributions are insignificant.

5.1.5 A Simple Model of Neutrino Mass

With these contributions to the neutrino mass matrix, how do we ensure that our model is consistent with the results in Equation 127? In this introductory model, we will assume that the neutrino mixing matrix U, generated from the experimental results, is approximately tribimaximal, meaning that [41]

$$\theta_{13} = 0, \qquad \sin^2 2\theta_{23} = 1, \qquad \tan^2\theta_{12} = 1/2. \tag{141}$$

The tribimaximal mixing matrix, and thus the neutrino mixing matrix, is

$$U = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}; \tag{142}$$

the three columns of this matrix roughly resemble the meson nonet [41]. We use this matrix to diagonalize the neutrino mass matrix, given by

$$\begin{aligned} m_\nu &= [m_\nu]_{ij}^{\mu\mu} + [m_\nu]_{ij}^{\lambda\lambda} + [m_\nu]_{ij}^{\lambda'\lambda'} \\ &\simeq \frac{\cos^2\beta}{\bar{m}}\mu_i\mu_j + \sum_{l,k}\frac{1}{8\pi^2}\lambda_{ilk}\lambda_{jkl}\frac{m_{\ell_l}m_{\ell_k}}{\bar{m}} + \sum_{l,k}\frac{1}{8\pi^2}\lambda'_{ilk}\lambda'_{jkl}\frac{m_{d_l}m_{d_k}}{\bar{m}} \end{aligned} \tag{143}$$

in this simple model [39]. Therefore, we have $\mathrm{diag}(m_1, m_2, m_3) = U^{-1}m_\nu U$ and can reverse this equation to find an expression for the mass matrix in terms of its eigenvalues:

$$\begin{aligned} m_\nu &= U\mathrm{diag}(m_1, m_2, m_3)U^{-1} \\ &= \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \times \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \times \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{3}(2m_1 + m_2) & \frac{1}{3}(-m_1 + m_2) & \frac{1}{3}(-m_1 + m_2) \\ \frac{1}{3}(-m_1 + m_2) & \frac{1}{6}(m_1 + 2m_2 + 3m_3) & \frac{1}{6}(m_1 + 2m_2 - 3m_3) \\ \frac{1}{3}(-m_1 + m_2) & \frac{1}{6}(m_1 + 2m_2 - 3m_3) & \frac{1}{6}(m_1 + 2m_2 + 3m_3) \end{pmatrix}. \end{aligned} \tag{144}$$

By setting the result in Equation 144 equal to Equation 143, we can find the values of the R-parity violating couplings at the weak scale that generate accurate neutrino masses and mixing angles [39]. In the models that we will study in the next sections, we will not assume the tribimaximal form of the neutrino mass matrix as recent results have proven that θ_{13} is nonvanishing.

5.2 Bilinear Models

With the general characteristics of the contributions established, we examine several specific theories of neutrino mass. The minimal extension of the MSSM involves only bilinear interactions,[18] and such a model can also be applied to theories with split supersymmetry. The bilinear terms can come from either the superpotential or the soft supersymmetry breaking sector, and results in one massive

[18] Trilinear-only theories are not a natural choice as trilinear $\not{R}_p$ will generate loop-level bilinear terms anyway, so we will neglect them in this paper [6].

neutrino from the tree-level terms and the other two neutrinos gaining mass from radiative corrections to the neutrino-neutralino mass matrix [6]. The bilinear-only theory is subject to some fine-tuning, but this problem can be resolved by implementing flavor symmetries, as we shall see in the next section.

5.2.1 Bilinear $\not{R}_p$ in the MSSM

The most basic, minimal extension of the MSSM, called bilinear R-Parity violation (BRPV), contains only one bilinear term in the superpotential:

$$W = W_{MSSM} + \epsilon_i \hat{L}_i \hat{H}_u, \tag{145}$$

where the $\epsilon_i = (\epsilon_e, \epsilon_\mu, \epsilon_\tau)$ parameters break lepton number [9]. This parameter has units of mass and must be small ($\epsilon_i \ll m_W$) in order to generate appropriate neutrino physics, but there is no reason *a priori* that this should be the case [19]. This problem can be resolved if we implement a new symmetry, such as horizontal family symmetry, or assume that R-parity is violated spontaneously, where $\epsilon =$ Yukawa coupling $\times$ sneutrino v.e.v. [32].

The BRPV also contains new soft supersymmetry breaking terms:

$$\mathcal{L}_{soft}^{BRPV} = B_{\epsilon_i} \tilde{L}_i H_u \tag{146}$$

where $B_{\epsilon_i} = B\epsilon_i$ has units of mass [9]. The two couplings ϵ_i and B_{ϵ_i} induce sneutrino v.e.v.s proportional to ϵ, which is why we required ϵ to be so small [9]. As previously discussed, bilinear terms also induce neutrino and neutralino mixing, resulting in the 7×7 neutral fermion mass matrix, which we will recast in the $\left\{-i\tilde{B}, -i\tilde{W}_3, \tilde{H}_u, \tilde{H}_d, \nu_\alpha\right\}$ basis as

$$M_N = \begin{pmatrix} M_{\chi^0} & m^\top \\ m & 0 \end{pmatrix} \tag{147}$$

in order to see the seesaw structure. In the new basis, the neutralino mass matrix is

$$M_{\chi^0} = \begin{pmatrix} M_1 & 0 & -\frac{1}{2}g'v_d & \frac{1}{2}g'v_u \\ 0 & M_2 & \frac{1}{2}gv_d & -\frac{1}{2}gv_u \\ -\frac{1}{2}g'v_d & \frac{1}{2}gv_d & 0 & -\mu \\ \frac{1}{2}g'v_u & \frac{1}{2}gv_u & -\mu & 0 \end{pmatrix} \tag{148}$$

and the m matrix characterizes the R-parity breaking in the theory:

$$m = \begin{pmatrix} -\frac{1}{2}g'v_1 & \frac{1}{2}gv_1 & 0 & \epsilon_1 \\ -\frac{1}{2}g'v_2 & \frac{1}{2}gv_2 & 0 & \epsilon_2 \\ -\frac{1}{2}g'v_3 & \frac{1}{2}gv_3 & 0 & \epsilon_3 \end{pmatrix}, \tag{149}$$

where v_i are the v.e.v.s of the sneutrinos [32]. Because ϵ_i is small, $M_{\chi^0} > m$ and Equation 147 closely resembles Equation 69. Using the seesaw mechanism as in Section 3.1.2, we find the effective light neutrino mass matrix to be $m_\nu^0 = -mM_{\chi^0}^{-1}m^\top$. We can expand this matrix to

$$\left[m_\nu^0\right]_{ij} = \frac{M_1 g^2 + M_2 g'^2}{4 \det\left(M_{\chi^0}\right)} \Lambda_i \Lambda_j \tag{150}$$

where $\Lambda_i = \mu v_i + v_d \epsilon_i$ [19]. This parameter Λ_i is called the alignment vector and is related to ξ [6]. The neutrino masses are the eigenvalues of m_ν^0 and, as we found in the general tree-level case, there is only one non-zero neutrino mass.

We can also gain some information on the mixing angles. The effective neutrino mass matrix can be diagonalized by a matrix V_ν as $V_\nu^\top m_\nu^0 V_\nu$, where

$$V_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & -\sin\theta_{23} \\ 0 & \sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \times \begin{pmatrix} \cos\theta_{13} & 0 & -\sin\theta_{13} \\ 0 & 1 & 0 \\ \sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix}; \tag{151}$$

we can then find the mixing angles to be

$$\tan\theta_{13} = \frac{\Lambda_e}{\sqrt{\Lambda_\mu^2 + \Lambda_\tau^2}} \tag{152}$$

$$\tan\theta_{23} = -\frac{\Lambda_\mu}{\Lambda_\tau} \tag{153}$$

in terms of the alignment vector [32]. From the experimental results, we know $\theta_{23} \sim 45°$ so $\Lambda_\mu \simeq \Lambda_\tau$, and since θ_{13} is small, $\Lambda_e < \Lambda_\mu, \Lambda_\tau$.

Since we only get one neutrino mass from the tree level matrix m_ν^0, the next-most massive neutrino arises at the one-loop level. These corrections to m_ν^0 are, schematically,

$$\left[m_\nu^1\right]_{ij} \simeq a^{(1)} \Lambda_i \Lambda_j + b^{(1)} \left(\Lambda_i \epsilon_j + \Lambda_j \epsilon_i\right) + c^{(1)} \epsilon_i \epsilon_j \tag{154}$$

where the coefficients are functions of supersymmetric parameters [9]. More specifically, we find

$$\left[m_\nu^1\right]_{ij} = \frac{1}{2}\left[\tilde{\Pi}_{ij}^\nu m_i^2 + \tilde{\Pi}_{ij} m_j^2\right] - \frac{1}{2}\left[M_{\chi_i^0} \tilde{E}_{ij}^\nu m_i^2 + M_{\chi_j^0} \tilde{E}_{ij}^\nu m_j^2\right] \tag{155}$$

where $\tilde{\Pi}$ and $\tilde{E}$ are the renormalized self-energies [32]. This new mass matrix m_ν^1 generates an additional non-zero eigenvalue, giving mass to the second neutrino. This mass m_{ν_2} has contributions from many types of loops, but the bottom-sbottom loop contributes most significantly [32]. Simplifying the mass equation further, we find that

$$m_{\nu_2} \simeq \frac{3}{16\pi^2} \sin(2\theta_{\bar{b}}) m_b \Delta B_0^{\bar{b}_2\bar{b}_1} \frac{\tilde{\epsilon}_1^2 + \tilde{\epsilon}_2^2}{\mu^2}, \tag{156}$$

where we have only taken into account the self-energy terms proportional to $\tilde{\epsilon}_i \times \tilde{\epsilon}_j$ [32]. In this equation, B_0 is a Passarino-Veltman function, m_b is the bottom quark mass, and $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$ are functions of ϵ_i and Λ_i [32]. The third neutrino remains massless in this model.

With the loop-level mass matrix, we can now find an expression for the third mixing angle. In the basis where the tree-level mass matrix m_ν^0 is diagonal, the one-loop level corrected mass matrix is

$$m_\nu^1 = \begin{pmatrix} c_1\tilde{\epsilon}_1\tilde{\epsilon}_1 & c_1\tilde{\epsilon}_1\tilde{\epsilon}_2 & c_2\tilde{\epsilon}_1\tilde{\epsilon}_3 \\ c_1\tilde{\epsilon}_2\tilde{\epsilon}_1 & c_1\tilde{\epsilon}_2\tilde{\epsilon}_2 & c_2\tilde{\epsilon}_2\tilde{\epsilon}_3 \\ c_1\tilde{\epsilon}_3\tilde{\epsilon}_1 & c_1\tilde{\epsilon}_3\tilde{\epsilon}_2 & c_0|\Lambda_i|^2 + c_2\tilde{\epsilon}_3\tilde{\epsilon}_3 \end{pmatrix} + \dots \tag{157}$$

where the coefficients c are functions of supersymmetric parameters and we ignore less significant terms [32]. We can approximate this matrix by defining

$$x \equiv \frac{c_1|\tilde{\epsilon}_i|^2}{c_0|\Lambda_i|^2} \ll 1; \tag{158}$$

therefore, the mass matrix becomes

$$m_\nu^1 = c_0|\Lambda_i|^2 \begin{pmatrix} x(\tilde{\epsilon_1}\tilde{\epsilon_1})/|\epsilon_i|^2 & x(\tilde{\epsilon_1}\tilde{\epsilon_2})/|\epsilon_i|^2 & x(\tilde{\epsilon_1}\tilde{\epsilon_3})/|\epsilon_i|^2 \\ x(\tilde{\epsilon_2}\tilde{\epsilon_1})/|\epsilon_i|^2 & x(\tilde{\epsilon_2}\tilde{\epsilon_2})/|\epsilon_i|^2 & x(\tilde{\epsilon_2}\tilde{\epsilon_3})/|\epsilon_i|^2 \\ x(\tilde{\epsilon_3}\tilde{\epsilon_1})/|\epsilon_i|^2 & x(\tilde{\epsilon_3}\tilde{\epsilon_2})/|\epsilon_i|^2 & 1 + x(\tilde{\epsilon_3}\tilde{\epsilon_3})/|\epsilon_i|^2 \end{pmatrix}. \tag{159}$$

The mass matrix can now be diagonalized by $\tilde{V}_\nu^\top m_\nu^1 \tilde{V}_\nu$ and, from this equation, we find that

$$\tan\theta_{12} = \frac{\tilde{\epsilon}_1}{\tilde{\epsilon}_2}; \tag{160}$$

this result is more approximate than the other mixing angle equations as we are only considering bottom-sbottom loops [32]. In terms of the original parameters ϵ_i and Λ_i, the mixing angle is

$$\tan\theta_{12} = \frac{\left[\epsilon_e\left(\Lambda_\mu^2 + \Lambda_\tau^2\right) - \Lambda_e\left(\Lambda_\mu\epsilon_\mu + \Lambda_\tau\epsilon_\tau\right)\right] / \sqrt{\left(\Lambda_\mu^2 + \Lambda_\tau^2\right)\left(\Lambda_e^2 + \Lambda_\mu^2 + \Lambda_\tau^2\right)}}{\left[\Lambda_\tau\epsilon_\mu - \Lambda_\mu\epsilon_\tau\right] / \sqrt{\Lambda_\mu^2 + \Lambda_\tau^2}}. \tag{161}$$

Clearly, this expression is much more complex than for the other two mixing angles, but we can place some further restrictions on the parameters. We cannot have $\Lambda_\nu = \Lambda_\tau$ and $\epsilon_\mu = \epsilon_\tau$ else the denominator will be zero. In order to get the experimental value for θ_{23} we defined $\Lambda_\nu = \Lambda_\tau$, so care must be taken to avoid $\epsilon_\mu = \epsilon_\tau$ as well. In addition, since we have observed $\theta_{12} \sim 30°$, we see that $\tilde{\epsilon}_1^2 \sim 3\tilde{\epsilon}_2^2$.

5.2.2 Bilinear $\not{R}_p$ in Split Supersymmetry

The simple bilinear model can also be applied to theories beyond the MSSM with similar results. As discussed in Section 4.5, split supersymmetry imposes an inverse hierarchy where the first and second generations experience supersymmetry breaking more strongly than the third generation and Higgs sector. As a result, split supersymmetry is a high-scale theory with heavy scalars (except the Higgs boson) and light fermions [15]. The theory has several advantages, but is not very natural given the arbitrary split between the generations. However, the hope is that the addition of R-parity violating terms generates accurate neutrino results and increases the naturalness of the theory.

The tree-level bilinear $\not{R}_p$ contribution proceeds in much the same way as in the MSSM. We begin by adding the $\not{R}_p$ terms to the split supersymmetry Lagrangian:

$$\mathcal{L}^{SS}_{\not{R}_p} = \epsilon_i \tilde{H}_u^\top i\sigma_2 L_i - \frac{1}{\sqrt{2}} a_i H^\top i\sigma_2 \left(-\tilde{g}_d \sigma \tilde{W} + \tilde{g}'_d \tilde{B}\right) L_i + \text{h.c.} \tag{162}$$

where $\tilde{g}$ are the split supersymmety Higgs-higgsino-gaugino couplings [17, 20]. After the Higgs boson gains its v.e.v. and ignoring terms that are irrelevant to neutrino mass, we can rewrite Equation 162 as

$$\mathcal{L}^{SS}_{\not{R}_p} = -\left[\epsilon_i \tilde{H}_u^0 + \frac{1}{2} a_i \left(\tilde{g}_d \tilde{W}_3 - \tilde{g}'_d \tilde{B}\right)\right] \nu_i + \text{h.c.} + \ldots, \tag{163}$$

where v is the v.e.v. of the Standard Model-like Higgs field [20]. As before, neutrino-neutralino mixing is induced and we can write the resulting mass matrix as

$$M_N^{SS} = \begin{pmatrix} M_{\chi^0}^{SS} & \left(m^{SS}\right)^\top \\ m^{SS} & 0 \end{pmatrix} \tag{164}$$

where the neutralino mass matrix is

$$M_{\chi^0}^{SS} = \begin{pmatrix} M_1 & 0 & -\frac{1}{2}\tilde{g}'_d v & \frac{1}{2}\tilde{g}'_u v \\ 0 & M_2 & \frac{1}{2}\tilde{g}_d v & -\frac{1}{2}\tilde{g}_u v \\ -\frac{1}{2}\tilde{g}'_d v & \frac{1}{2}\tilde{g}_d v & 0 & -\mu \\ \frac{1}{2}\tilde{g}'_u v & \frac{1}{2}\tilde{g}_u v & -\mu & 0 \end{pmatrix} \tag{165}$$

and the mixing matrix is

$$m^{SS} = \begin{pmatrix} -\frac{1}{2}\tilde{g}'_d a_1 v & \frac{1}{2}\tilde{g}'_d a_1 v & 0 & \epsilon_1 \\ -\frac{1}{2}\tilde{g}'_d a_2 v & \frac{1}{2}\tilde{g}'_d a_2 v & 0 & \epsilon_2 \\ -\frac{1}{2}\tilde{g}'_d a_3 v & \frac{1}{2}\tilde{g}'_d a_3 v & 0 & \epsilon_3 \end{pmatrix}; \tag{166}$$

these matrices are in the same basis as before: $\left\{-i\tilde{B}, -i\tilde{W}_3, \tilde{H}_u, \tilde{H}_d, \nu_\alpha\right\}$ [17]. The neutrino mass matrix is then, of course, $m_\nu^{SS} = -m^{SS}\left(M_{\chi^0}^{SS}\right)^{-1}\left(m^{SS}\right)^\top$, which can be written as

$$\left[m_\nu^{SS}\right]_{ij} = v^2 \frac{M_1 \tilde{g}_d^2 + M_2 \tilde{g'}_d^2}{4 \det\left(M_{\chi^0}^{SS}\right)} \lambda_i \lambda_j \tag{167}$$

where $\lambda_i \equiv a_i \mu + \epsilon_i$ and is related to Λ_i as $\Lambda_i = \lambda_i v_d$ [17, 20]. Since this result is similar in form to Equation 150, it is no surprise that we again get only one massive neutrino from the tree-level contribution.

In the MSSM case, the bilinear loop contributions were considered next, but in split supersymmetry, we shall see that these contributions are negligible. In this theory, the one-loop contributions contain a factor of

$$\eta_i \frac{m_Z^2}{m_{L_i}^2}, \tag{168}$$

where m_Z is the mass of the Z-boson, m_{L_i} is the slepton mass (on the order of the split supersymmetry scale m_S) and η_i measures the mixing between the sleptons and Higgs bosons [15]. While η_i usually is large enough to produce significant contributions at loop level, in split supersymmetry m_S can be on the order of $10^9 - 10^{13}$ GeV [15]. The Z-boson mass remains at around 90 GeV and so the term in Equation 168 becomes negligible, no matter the value of η_i.

To see another reason why we need more than just bilinear loop terms, we will examine the one-loop contributions. In the MSSM the Higgs bosons mix with the sneutrinos, but in split supersymmetry the sneutrinos are much heavier than the light Higgs boson, and so they decouple. As a result, the only important contributions are the loops involving the neutralinos and the light Higgs, as pictured in Figure 11. The couplings in this diagram, unlike their MSSM equivalents, involve the matrix $\mathcal{N}$, which is 7×7 and diagonalizes M_N in Equation 147, and replaces the Higgs mixing angle with $\alpha = \beta - \pi/2$ as it is decoupled [20]. The contribution from the Higgs boson has the form

$$\Delta[m_\nu^1]_{ij} \sim A^h \lambda_i \lambda_2 \tag{169}$$

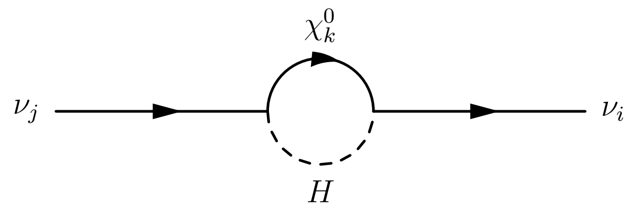


Figure 11: Loop contribution to neutrino mass from neutralinos and light Higgs bosons [20].

where A^h involves the couplings; the other insignificant contributions have the same form [20]. This type of contribution merely renormalizes the tree level neutrino mass matrix and does not break its symmetry, so mass is *not* generated for all the neutrinos [20]. Therefore, in order to generate mass for the two lighter neutrinos, as desired at the loop level, we must have contributions from non-bilinear loop terms.

5.3 Flavor Symmetry Models

Recall that in our general exploration of tree- and loop-level contributions, the $\not{R}_p$ couplings are not restricted, but we can constrain the couplings with flavor symmetries [6]. This additional symmetry, while not entirely natural, can allow BRPV models to generate appropriate neutrino results.

5.3.1 $A_4 \times Z_2$ Symmetry Model

The first flavor symmetry model proposed, in Ref. [9], adds an $A_4 \times Z_2$ symmetry and singlet superfield[19] $\hat{S}$ to the MSSM. The Z_2 symmetry serves to forbid all $\not{R}_p$ interactions except the $\epsilon \hat{L} \hat{H}_u$ bilinear term needed for BRPV and the A_4 symmetry, since all MSSM fields are triplet representations in it, reduces the number of BRPV parameters to one [9]. We define a new v.e.v. alignment for the Higgs and left-handed sneutrinos because of this new triplet structure:

$$\langle H_u \rangle = \frac{1}{\sqrt{2}} \left(v_{u_1}, v_{u_2}, v_{u_3}\right) = v_{u_3} \left(r^u, -1, 1\right) \tag{170}$$

$$\langle H_d \rangle = \frac{1}{\sqrt{2}} \left(v_{d_1}, v_{d_2}, v_{d_3}\right) = v_{d_3} \left(r^d, -1, 1\right) \tag{171}$$

$$\langle \tilde{\nu}_L \rangle = \frac{1}{\sqrt{2}} \left(v_{L_e}, v_{L_\mu}, v_{L_\tau}\right) = v_{L_\tau} \left(a^\nu, -1, 1\right), \tag{172}$$

where r and a are parameters in the fermion mass matrix [9]. We also redefine $\Lambda_i = \mu v_{L_i} + v_{d_i}\epsilon$; there is now only a single bilinear parameter ϵ instead of three ϵ_i

[19]Like in the NMSSM.

because of the flavor symmetry [9]. As a result, we find

$$\begin{aligned}\Lambda_\mu &= \mu v_{L_\mu} + v_{d_2}\epsilon \\ &= \mu(-1)v_{L_\tau} + (-1)v_{d_3}\epsilon \\ &= -\left(\mu v_{L_\tau} + v_{d_3}\epsilon\right) \\ &= -\Lambda_\tau \end{aligned} \tag{173}$$

which is a condition we wanted to impose on Equation 153 for an accurate θ_{23} mixing angle. As opposed to the plain BRPV theory, this condition is *required*, not merely suggested, in the flavor theory.

To see that the flavor symmetry BRPV theory continues to generate appropriate neutrino values, we find the neutrino mass matrix to be, schematically,

$$m_\nu = \begin{pmatrix} c+\alpha(2b+\alpha a) & c+b(\alpha-1)-\alpha a & b+c+\alpha(a+b) \\ c+b(\alpha-1)-\alpha a & a-2b+c & c-a \\ b+c+\alpha(a+b) & c-a & a+2b+c \end{pmatrix} \tag{174}$$

where we have defined

$$\begin{aligned}\Lambda_\mu &= \Lambda_\tau = \Lambda \\ \Lambda_e &= \alpha\Lambda \\ a &= \left(a^{(0)} + a^{(1)}\right)\Lambda^2 \\ b &= b^{(1)}\Lambda\epsilon \\ c &= c^{(1)}\epsilon^2 \end{aligned} \tag{175}$$

as our parameters [9]. The $a^{(0)}$ parameter is equal to the coefficient in Equation 150 and $a^{(1)}, b^{(1)}, c^{(1)}$ are the same as in Equation 154. We see that when the one-loop contributions are $b = c = 0$, m_ν reduces to the tree-level m_ν^0 with only one non-zero eigenvalue, $m_{\nu_3} = a|\Lambda_i|^2 = a(2+\alpha^2)$ with an eigenvector along the $(\alpha, 1, -1)$ direction [9]. We can assume that $b, c \ll a$ (the loop parameters are smaller than the tree-level parameter) which generates the phenomenological result $m_{\nu_2} \ll m_{\nu_3}$.

5.3.2 Z_3 Symmetry Model

The $A_4 \times Z_2$ flavor symmetry, though it only involves bilinear terms, adds some unwanted complexity to the scalar sector. Therefore, a simpler symmetry, Z_3, is proposed in Ref. [46] by Peinado and Vicente, which eliminates $\hat{S}$ and adds trilinear $\not{R}_p$ terms.[20] Since we argued in the previous section that these trilinear terms

[20] Other formulation of Z_3 models have been proposed but this model accommodates the most recent neutrino results, specifically a non-vanishing θ_{13}.

Field	$\hat{L}_1$	$\hat{L}_2$	$\hat{L}_3$	$\hat{E}_1$	$\hat{E}_2$	$\hat{E}_3$	$\hat{H}_u$	$\hat{H}_d$
$\mathbf{Z_3}$	1	ω	ω^2	1	ω^2	ω	1	1

Table 7: The Z_3 charges of the MSSM superfields, where $\omega = e^{i2\pi/3}$.

must be included (in the absence of additional superfields) to generate all neutrino masses, this solution seems more natural while also inducing the appropriate neutrino masses and mixings.

Under the Z_3 symmetry, we assign the charges in Table 7 to the MSSM superfields [46]. This particular form of the charges leads to a diagonal mass matrix for the charged leptons. In addition, the quark superfields are singlets under Z_3 so that the only non-zero quark couplings in the superpotential are λ_{122}, λ_{133} and λ_{231} [46]. The relevant lepton terms in the superpotential are

$$W_{Z_3} = \epsilon \hat{L}_1 \hat{H}_u + Y_e^i \hat{E}_i \hat{L}_i \hat{H}_d + \lambda_{ijk} \hat{L}_i \hat{L}_j \hat{E}_k + \lambda'_{jk} \hat{Q}_j \hat{L}_1 \hat{d}_k. \qquad (176)$$

Note that, as in the $A_4 \times Z_2$ flavor symmetry theory, we only have a single ϵ parameter. In the Z_3 model, baryon number conservation must be imposed as we cannot have both baryon and lepton number violation, but by allowing trilinear terms we do not automatically forbid the baryon number violating terms. The Z_3 symmetry is softly broken in the scalar potential, and the relevant terms are of the form

$$V_{soft} \sim m^2_{L_i H_d} \tilde{L}_i^* H_d + B_\epsilon \tilde{L}_i H_u, \qquad (177)$$

where $m^2_{L_i H_d}$ and B_ϵ are generation-dependent couplings that we will assume are unified at the grand unification scale [46]. Importantly, the generational coupling equality does not hold at $\bar{m}$, where the third generation Yukawa couplings are larger than the other two due to the renormalization group equations [46]. Note that this pattern, except inverted, also occurs in split supersymmetry. This coupling inequality is essential for generating mixing between the second and third neutrino generations.

Another important assumption is that, as in the BRPV model, our single ϵ parameter is small. In the BRPV model, the tree-level mass matrix is proportional to this parameter, which effectively restricts the single neutrino mass to be appropriately small as the loop level contributions only affect the other two neutrino masses. However, in the Z_3 model we have additional bilinear soft terms that contribute significantly at the loop level, rendering the tree-level bilinear contributions negligible.

We will therefore discuss the bilinear loop level contributions first. Relatively large $m^2_{L_i H_d}$ and B_ϵ couplings are required in order to get suitably large neutrino masses; if the loop contributions are too small, then the sneutrino v.e.v.s and thus the tree-level contributions are too large [46]. To keep the sneutrino v.e.v.s small, we must impose some fine-tuning so that the loop-level contributions to them cancel out:

$$m^2_{L_iH_d} v_d + B_\epsilon v_u \simeq 0, \tag{178}$$

which implies the relation

$$m^2_{L_iH_d} \simeq \tan\beta B_\epsilon \tag{179}$$

between the loop-level parameters [46]. By defining the lepton number violation scale as $m^2_{L_iH_d}, B_\epsilon \sim m^2_{\not{R}_p}$, we write Equation 137 as

$$\left[m^1_\nu\right]^{BRPV}_{ij} \sim \frac{g^2}{16\pi^2} \frac{m^2_{\not{R}_p} \cdot m^2_{\not{R}_p}}{\bar{m}^3} \tag{180}$$

where the β dependence is included in $m^2_{\not{R}_p}$ [46]. In Section 5.1.3 we noted that the only suppression on this contribution was the value of the parameter. Because the parameters $m^2_{L_iH_d}$ and B_ϵ are large in this model, this loop contribution is significant.

Because the Z_3 symmetry model insists on a splitting between the third and first and second generations in the $m^2_{L_iH_d}$ and B_ϵ parameters, we must take this deviation into account when constructing the neutrino mass matrix. Therefore, we rewrite the bilinear loop level contribution as

$$\left[m^1_\nu\right]^{BRPV}_{ij} = a \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + d \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 2 \end{pmatrix}, \tag{181}$$

where a is equal to the right-hand side of Equation 180 and d is a similar equation that replaces the parameter $m^2_{\not{R}_p}$ with $m^2_{\not{R}_p} + \delta m^2$ [46]. The term δm^2 measures the third generation's deviation from universality and its effects are shown in the d-matrix in $[m^1_\nu]^{BRPV}_{ij}$.

Next we consider the trilinear $\not{R}_p$ terms. The only allowed λ couplings are λ_{122}, λ_{133} and λ_{231}, which only contribute to the 11, 23 and 32 elements of the neutrino mass matrix [46]. In addition, the λ' couplings contribute to the 11 element since only $\hat{L}_1$ is included in that term. The neutrino mass contributions therefore look like

$$\left[m^1_\nu\right]^{TRPV}_{ij} = \begin{pmatrix} b & 0 & 0 \\ 0 & 0 & c \\ 0 & c & 0 \end{pmatrix} \tag{182}$$

where c is from the λ terms and b is from the λ' terms [46]. We find, using Equation

Coupling	Bound	Source
λ_{122}	2.7×10^{-2}	Neutrino masses
λ_{133}	1.6×10^{-3}	Neutrino masses
λ_{231}	0.05	Leptonic τ decay
$\lambda'_{jk}(j \neq k)$	$0.02 - 0.47$	Various processes
$\lambda'_{jk}(j = k)$	$3.3 \times 10^{-4} - 0.02$	$\beta\beta 0\nu$ decay, neutrino masses

Table 8: The experimental bounds on loop-level couplings [6, 46].

135, that

$$\left[m_\nu^{TRPV}\right]_{23}^{\lambda\lambda} = c \sim \frac{1}{16\pi^2}\lambda_{231}^2 \mu \tan\beta \frac{m_e m_\tau}{\bar{m}} \tag{183}$$

and, from Equation 136,

$$\left[m_\nu^{TRPV}\right]_{11}^{\lambda'\lambda'} = b \sim \sum_{l,k} \frac{3}{8\pi^2}\lambda'_{1lk}\lambda'_{1kl}\mu \tan\beta \frac{m_{d_l} m_{d_k}}{\bar{m}}. \tag{184}$$

Since the masses of quarks and leptons are experimentally known and the λ coupling bounds are given in Table 8, the neutrino masses depend mostly on the μ and $\tan\beta$ parameters. For example, mass contributions to c on the order of 0.1 eV are given by $\mu \sim \bar{m}$ and $\tan\beta = 10$ [46].

The total mass contribution from the bilinear and trilinear $\not{R}_p$ loops is found by summing Equations 181 and 182:

$$m_\nu = \left[m_\nu^1\right]_{ij}^{BRPV} + \left[m_\nu^1\right]_{ij}^{TRPV} = \begin{pmatrix} a+b & a & a+d \\ a & a & a+c+d \\ a+d & a+c+d & a+2d \end{pmatrix}. \tag{185}$$

Note that d, which measures the third generation's splitting from the other two at the SUSY scale, contributes to both the θ_{23} deviations from maximal and, simultaneously, θ_{13} deviations from zero [46]. This correlation between the two mixing angles can be confirmed with numerical studies as in Figure 12. The different bands of results correspond to different CP branches [46]. The intrinsic CP charge of the neutrinos has four possible cases, denoted η_i: $\eta_1 = (-,+,+)$, $\eta_2 = (+,-,+)$, $\eta_3 = (+,+,-)$, and $\eta_1 = (+,+,+)$ [46]. From the results in Figure 12, we see that the η_2 case seems to be preferred.

The Z_3 flavor symmetry model can accommodate the most recent neutrino mass and mixing results, but does rely on introducing a new symmetry, which is as physically unmotivated as the R-parity symmetry it is meant to replace. In addition, baryon number conservation is manually forbidden instead of a natural

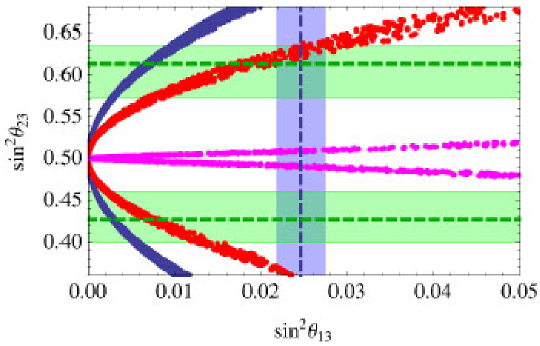


Figure 12: A plot of $\sin^2\theta_{23}$ as a function of $\sin^2\theta_{13}$. The dashed lines are the best-fit values while the shaded areas correspond to allowed regions. The blue dots are for the η_1 CP branch, red dots for η_2 and pink for η_3 (no points for the η_4 case) [46].

consequence of the theory. The last two sections will examine theories where any additional symmetries are accidental.

5.4 Models with Additional Fields

Instead of imposing a flavor symmetry, adding sterile neutrino fields to the MSSM is the third general way of generating neutrino mass in $\not{R}_p$ theories. This method was introduced in Section 3.1.2 to give light neutrinos mass via the seesaw mechanism; in this section, we will offer some justification as to why and how these new fields can be added. In both cases, the additional fields also serve to solve a problem within supersymmetry.

5.4.1 Minimal Flavor Violation

We can modify the MSSM by requiring minimal flavor violation (MFV), which in turn generates R-parity violation and neutrino mass. Minimal flavor violation is a principle rather than a strict symmetry, but has the same desired result of constraining the MSSM to phenomenological limits.

While supersymmetry has several attractive features, such as the solution to the gauge hierarchy problem, gauge coupling unification and a dark matter candidate, the generic theory also allows for some undesirable flavor and CP violating effects [45]. A natural solution to the flavor problem is the idea of minimal flavor violation, which states that all flavor and CP-violating interactions arise from the known, Standard Model Yukawa couplings [3]. The MSSM needs additional assumptions to be made "phenomenologically acceptable:" the usual assumptions are R-parity and flavor universality [18]. We have been using flavor universality throughout this paper, which asserts that all the soft symmetry-breaking masses are flavor universal at a scale m_{GUT} and the coupling constants A in Equation 112 are proportional to the λ couplings in the superpotential. We can replace these two assumptions with one: minimal flavor violation.

Minimal flavor violation can be imposed on the Standard Model as well as the MSSM, so we will examine this simpler case first. As we know, the Standard Model fermions are in two $SU(2)_L$ doublets, Q_m and L_m, and three singlets, u_m, d_m and e_m. The global flavor symmetry of the Standard Model can be written as

$$G_F \equiv SU(3)_q^3 \times SU(3)_\ell^2 \times U(1)_B \times U(1)_L \times U(1)_Y \times U(1)_{PQ} \times U(1)_{E_R} \quad (186)$$

where the five $U(1)$ charges are associated with baryon number B, lepton number L, hypercharge Y, the Peccei-Quinn symmetry present in models with two Higgs doublets and the global rotation of the $SU(2)_L$ singlet [3]. The $SU(3)$ groups have the form

$$SU(3)_q^3 = SU(3)_{Q_m} \times SU(3)_{u_m} \times SU(3)_{d_m} \quad (187)$$
$$SU(3)_\ell^3 = SU(3)_{L_m} \times SU(3)_{e_m}; \quad (188)$$

note that these are $SU(3)$ because each describes the three generations of particles [3]. The group $SU(3)_q^3 \times SU(3)_\ell^2 \times U(1)_{PQ} \times U(1)_{E_R}$ is broken in the Standard Model by the Yukawa interactions; for minimal flavor violation, these are the *only* interactions that break the flavor symmetry [18]. We introduce the spurion fields Y_u, Y_d and Y_e, which transform as

$$Y_u \sim (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1})_{SU(3)_q^3}, \qquad Y_d \sim (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1})_{SU(3)_q^3}, \qquad Y_e \sim (\mathbf{3}, \bar{\mathbf{3}})_{SU(3)_\ell^2}, \quad (189)$$

in order to recover flavor invariance [3]. The background values of these fields can be rotated into

$$Y_u = V^\dagger \lambda_u, \qquad Y_d = \lambda_d, \qquad Y_L = \lambda_\ell, \quad (190)$$

where the λ terms are diagonal and V is the CKM matrix [3]. Recall that the CKM matrix controls the mixing between the quark generations and has the form

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{-i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{-i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{-i\delta} & -c_{12}s_{23} - s_{12}c_{23}c_{13}e^{-i\delta} & c_{23}c_{13} \end{pmatrix}. \quad (191)$$

For minimal flavor violation, we require that all higher-dimension operators are constructed from Standard Model and Y spurion fields only and invariant under CP and G_F; the Yukawa couplings completely determine any flavor violation and the CKM phase δ completely determines any CP violation [3]. In supersymmetry, the Y fields appear in the MSSM superpotential Yukawa terms (see Equation 102), so we assign them to a chiral superfield representation; we expect that at the UV scale the spurions would manifest as the v.e.v.s of a heavy chiral superfield [18]. As a result, the conjugate couplings $Y^\dagger$ are not allowed in the superpotential, which greatly limits the number of possible terms. In addition, any allowed terms, including potentially problematic terms, are suppressed by the Yukawa couplings and CKM matrix. Therefore, the imposition of MFV solves the problems that both R-parity and flavor universality were meant to address.

However, all the allowed terms in the superpotential leave the lepton number symmetry $U(1)_L$ unbroken so we must introduce additional spurion fields, similar to Section 3.1.1, in order to generate neutrino mass [18]. The three additional sterile neutrinos N gain Majorana masses at a heavy scale M_R and generate mass for the left-handed neutrinos through the superpotential

$$W_N = Y_e L_m H_d \bar{e} + Y_N L_m H_u \bar{N} + \frac{1}{2} M_N \bar{N} \bar{N}, \tag{192}$$

where the sterile neutrino masses $M_N \sim M_R$ [18]. The new Yukawa coupling Y_N expands the lepton sector symmetry to $SU(3)_{L_m} \times SU(3)_{e_m} \times SU(3)_N$ and gives the left-handed neutrinos a Majorana mass on the order of $Y_N^2 v^2 / M_R$ [18]. Since M_R is large, the neutrino masses are small, as desired. The terms in W_N imply the presence of two new spurions: Y_N and M_N. Note that unlike the rest of the spurion Yukawa couplings, M_N has dimensions of mass and so we will expand it to

$$\mu_N \equiv \frac{1}{\Lambda_N} M_N, \tag{193}$$

where Λ_R is an unknown mass scale that satisfies $M_R \lesssim \Lambda_R$ and $\Lambda_R \gg m_{soft}$ [18].

We can find the flavor singlets involving the sterile neutrino parameters and, from these, construct only the $\lambda_{ijk} L_i L_j \bar{e}_k$ term in the lepton number-violating superpotential in Equation 108 [18]. However, the bilinear term $\epsilon L H_u$ term is also generated: though it is forbidden by the Z_3 lepton number symmetry present in MFV models, this symmetry is broken by M_N. The superpotential thus has the form

$$W_{LNV} = \frac{1}{2\Lambda_R} w LL \left(\tilde{Y}_N M_N \tilde{Y}_N \right) Y_e \bar{e} + m_{soft} \left(\mathcal{V}^\dagger \right)^a L_a H_u \tag{194}$$

where $\tilde{Y} \equiv (\det Y) Y^{-1}$, w is unknown and $\mathcal{V}$ is composed of the spurion fields [18]. The B term in the soft supersymmetry breaking sector is also generated because of lepton number symmetry breaking, yielding

$$\mathcal{L}_{soft}^{MFV} \supset m_{soft}^2 \left(\mathcal{V}^\dagger\right)^a \tilde{L}_a H_u + \text{h.c.} \tag{195}$$

which leads to the left-handed sneutrino gaining a v.e.v. of $\langle\tilde{\nu}\rangle \sim -v_u \mathcal{V}_a$ [18]. As before, the sneutrino v.e.v.s break lepton number and generate gaugino-Higgsino-lepton mixings, and so we have regained the effects of R-parity violation through MFV.

We can use the seesaw mechanism to generate neutrino mass by two different methods. The gaugino-lepton mixing term generated by the sneutrino v.e.v. is approximately $-v_u\lambda\left(\mathcal{V}^\dagger L\right) + \text{c.c.}$ and can contribute additional mass to the left-handed neutrinos via the seesaw mechanism [18]. This contribution is of the order

$$\delta m_\nu \sim \frac{\mathcal{V}^2 v_u^2}{m_\lambda}, \tag{196}$$

which we estimate to be on the order of 1 eV [18]. However, the imposition of realistic proton decays will place a strong bound on $\mathcal{V}$ to be small and so these seesaw contributions are negligible [18]. If we instead integrate out the heavy neutrinos and only consider a theory below the M_R scale, we find the only one term $Y_N M_N^{-1} Y_N^\top$ to be relevant for neutrino mass, giving a more restrictive theory for lepton number violation [18]. This term is analagous to the Type I seesaw formula in Equation 71 and so we can proceed as in Section 3.1.2 to find expressions for the light neutrino masses.

5.4.2 The $\mu\nu$SSM

Lastly, we will look at the $\mu\nu$SSM, which generates neutrino mass with the addition of right-handed neutrino superfields, similar to the previous section. Unlike the MFV model, though, the $\mu\nu$SSM is constructed to solve the μ problem instead of flavor or CP violation. Recall that the μ problem states that the supersymmetry-preserving parameter μ is correlated to a soft supersymmetry-breaking parameter, and it is also too small to account for the observed Higgs mass. Instead of relying on R-parity for an explanation, we will require that the v.e.v.s of some new fields generate the μ-term after symmetry breaking, hence the name "μ (terms) from ν (v.e.v.s) Supersymmetric Standard Model" [27]. We mentioned one solution in Section 3.3.3, the NMSSM, which adds only one field but fails to explain neutrino masses and mixings. The BRVP, while it can generate the correct results, only exacerbates the μ problem as it adds three new bilinear terms [43]. Therefore, we look to a new theory, the $\mu\nu$SSM, which introduces three sterile neutrinos $\hat{\nu}_i^c$ and provides an explanation for even the most recent neutrino observations.

As always, we define the theory by its superpotential first. The $\mu\nu$SSM adds three neutrinos, resulting in a new Yukawa term and two trilinear terms involving the sterile neutrinos and the Higgs fields:

$$\begin{aligned} W_{\mu\nu SSM} =&\epsilon_{ab}\left(Y_u^{ij}\hat{Q}_i^a\hat{H}_u^b\hat{u}_j^c + Y_d^{ij}\hat{H}_d^a\hat{Q}_i^b\hat{d}_j^c + Y_e^{ij}\hat{H}_d^a\hat{L}_i^b\hat{e}_j^c + Y_\nu^{ij}\hat{L}_i^a\hat{H}_u^b\hat{\nu}_j^c\right) \\ &-\epsilon_{ab}\lambda_i\hat{\nu}_i^c\hat{H}_d^a\hat{H}_u^b + \frac{1}{3}\kappa_{ijk}\hat{\nu}_i^c\hat{\nu}_j^c\hat{\nu}_k^c, \end{aligned} \tag{197}$$

where Y^{ij} are matrices, λ_i is a vector, and κ_{ijk} is a symmetric tensor [23]. The new Yukawa term generates Dirac masses for the neutrinos, $m_D^{ij} = Y_\nu^{ij}v_2$, and the terms in the second line of Equation 197 breaks lepton number and R-parity *explicitly*. Because of explicit lepton number breaking no majoron is generated after symmetry breaking, and because of explicit R-parity breaking the phenomenology of the $\mu\nu$SSM will be much different from the MSSM [40]. In addition, the κ term forbids an unwanted axion associated with a global $U(1)$ breaking [27]. The size of the breaking can be determined by realizing that in the $Y_\nu \to 0$ limit, the model is equal to the NMSSM with conserved R-parity (as the $\hat{\nu}^c$ are now merely singlet superfields) [43]. With the Y_ν nonzero, the $\hat{\nu}^c$ fields are right-handed neutrinos, thus breaking R-parity. This breaking generates an electroweak scale seesaw (explained later) and so $Y_\nu \lesssim 10^{-6}$ to generate correct neutrino masses; this value is acceptable as the electron coupling Y_e is on the same scale [43].

Note that the superpotential contains only trilinear terms; the MSSM μ term $\mu\hat{H}_u\hat{H}_d$ is not present. In order to exclude this term we impose a Z_3 symmetry on the theory, which is justified as the low-energy limit of string constructions[21], have this symmetry [40]. The Z_3 symmetry leads to a cosmological domain wall problem, but this issue can be resolved with small non-renormalizable operators that break the symmetry and thus undo the problematic degeneracy of the three original v.e.v.s [43]. However, as we mentioned in Section 5.2.1 trilinear $\not{R}_p$ terms generate loop-level bilinear terms, so this theory will inevitably have some bilinear terms.

After spontaneous electroweak symmetry breaking, the neutral scalar fields adopt the following vacuum expectation values:

$$\langle H_1^0\rangle = v_1, \qquad \langle H_2^0\rangle = v_2, \qquad \langle\tilde{\nu}_i\rangle = \nu_i, \qquad \langle\tilde{\nu}_i^c\rangle = \nu_i^c, \tag{198}$$

where $\tilde{\nu}$ are the scalar component of the neutrino superfields and we are in the electroweak (not mass) basis [40]. The v.e.v. of $\hat{\nu}^c$ generates the effective bilinear terms $\epsilon_i\hat{H}_u\hat{H}_i$ and $\mu\hat{H}_d\hat{H}_u$ (where the first is the BRPV bilinear term and the second is the MSSM μ term) and determines the coefficients: $\epsilon_i = Y_\nu^{ij}\nu_j^c$ and $\mu = \lambda_i\nu_i^c$ [58]. Because we expect ν^c to be at the electroweak scale and $\lambda \sim \mathcal{O}(1)$,

[21] These are relevant as supersymmetry breaking is mediated by gravity.

μ is at the electroweak scale, and we have therefore found a solution to the μ problem [27]. The parameter μ now has no correlation to the soft symmetry-breaking terms.

With the v.e.v.s, we now find the tree-level neutral scalar potential and the minimization conditions, eliminating the soft masses from the theory in favor of the v.e.v.s [43]. One of the minimization conditions is

$$\sum_j Y_\nu^{ij} v_2 \zeta^j + \gamma_g \xi_v \nu_i + r_c^i \eta + \sum_j \left(m_{\tilde{L}}^2\right)^{ji} \nu_j + \sum_j \left(A_\nu Y_\nu\right)^{ij} \nu_j^c v_2 = 0 \tag{199}$$

where ζ, γ, etc. are functions of the v.e.v.s and couplings [27]. Recall that Y_ν is very small in order to generate correct neutrino masses, and as $Y_\nu \to 0$, we see in Equation 199 that $\nu_i \to 0$. Therefore, ν_i must be small as well. By ignoring second-order terms of Y_ν and ν_i, we can solve Equation 199:

$$\nu_i \approx \left(\frac{Y_\nu^{ik} u_c^{kj} v_2 - \mu v_1 Y_\nu^{ij} + (A_\nu Y_\nu)^{ij} v_2}{\gamma_g (v_1^2 - v_2^2) + m_{\tilde{L}}^2} \right) \nu_j^c + \left(\frac{Y_\nu^{ij} \lambda^j v_1 v_2^2}{\gamma_g (v_1^2 - v_2^2) + m_{\tilde{L}}^2} \right) \tag{200}$$

where A_ν is a parameter of the theory, $u_c^{ij} = \sum_k \kappa^{ijk} v_k^c$ and $\gamma_g = (g_1^2 + g_2^2)/4$ [27]. From this equation, we see that the left-handed sneutrino v.e.v. can be nonzero even when the singlet sneutrinos have no v.e.v.s. However, the singlet sneutrino v.e.v.s are required in order to generate the μ terms, so the left-handed sneutrino v.e.v.s must depend on the singlet sneutrinos.

As before, the neutrinos and neutralinos mix in the $\mu\nu$SSM. The neutral fermion mass term in the $\mu\nu$SSM Lagrangian has the form

$$\mathcal{L}_{fermion} = -\frac{1}{2} \chi'^{0\top} M_N \chi'^{0} + \text{h.c.} \tag{201}$$

where $\chi'^{0\top} = \left(\tilde{B}^0, \tilde{W}^0, \tilde{H}_d, \tilde{H}_u, \nu_{R_i}, \nu_{L_i} \right)$ is the weak interaction basis [58]. The neutral fermion mass matrix M_N is now 10×10, with the four neutralinos, three left-handed neutrinos and three new sterile neutrinos, but has the same form as Equation 147:

$$M_N = \begin{pmatrix} M_{\chi_R^0} & m^\top \\ m & 0_{3\times 3} \end{pmatrix} \tag{202}$$

where $M_{\chi_R^0}$ is the neutralino mass matrix with the right-handed neutrinos and m

is the neutralino-neutrino mixing matrix [58]. These matrices are

$$M_{\chi_R^0} = \begin{pmatrix} M_1 & 0 & -\frac{g_1}{\sqrt{2}}v_d & \frac{g_1}{\sqrt{2}}v_u & 0 & 0 & 0 \\ 0 & M_2 & \frac{g_2}{\sqrt{2}}v_d & -\frac{g_2}{\sqrt{2}}v_u & 0 & 0 & 0 \\ -\frac{g_1}{\sqrt{2}}v_d & \frac{g_2}{\sqrt{2}}v_d & 0 & -\lambda_i \nu_i^c & -\lambda_1 v_u & -\lambda_2 v_u & -\lambda_3 v_u \\ \frac{g_1}{\sqrt{2}}v_u & -\frac{g_2}{\sqrt{2}}v_u & -\lambda_i \nu_i^c & 0 & y_1 & y_2 & y_3 \\ 0 & 0 & -\lambda_1 v_u & y_1 & 2\kappa_{11j}\nu_j^c & 2\kappa_{12j}\nu_j^c & 2\kappa_{13j}\nu_j^c \\ 0 & 0 & -\lambda_2 v_u & y_2 & 2\kappa_{21j}\nu_j^c & 2\kappa_{22j}\nu_j^c & 2\kappa_{23j}\nu_j^c \\ 0 & 0 & -\lambda_3 v_u & y_3 & 2\kappa_{31j}\nu_j^c & 2\kappa_{32j}\nu_j^c & 2\kappa_{33j}\nu_j^c \end{pmatrix} \tag{203}$$

and

$$m = \begin{pmatrix} -\frac{g_1}{\sqrt{2}}\nu_1 & \frac{g_2}{\sqrt{2}}\nu_1 & 0 & Y_\nu^{1i}\nu_i^c & Y_\nu^{11}v_u & Y_\nu^{12}v_u & Y_\nu^{13}v_u \\ -\frac{g_1}{\sqrt{2}}\nu_2 & \frac{g_2}{\sqrt{2}}\nu_2 & 0 & Y_\nu^{2i}\nu_i^c & Y_\nu^{21}v_u & Y_\nu^{22}v_u & Y_\nu^{23}v_u \\ -\frac{g_1}{\sqrt{2}}\nu_3 & \frac{g_2}{\sqrt{2}}\nu_3 & 0 & Y_\nu^{3i}\nu_i^c & Y_\nu^{31}v_u & Y_\nu^{32}v_u & Y_\nu^{33}v_u \end{pmatrix} \tag{204}$$

where $y_i = -\lambda_i v_d + Y_\nu^{ij}\nu^j$ [58].

Note that though the terms of $M_{\chi_R^0}$ are at the electroweak scale, the terms of m are small because of Y_ν and ν_i and the seesaw structure is preserved. In fact, the electroweak seesaw matrix is *always* present in this theory. With a purely Dirac mass for the neutrinos, we get $Y_\nu \sim 10^{-13}$, much smaller than the $Y_\nu \sim Y_e \sim 10^{-6}$ needed in a theory with an electroweak seesaw and the associated Majorana neutrino mass contributions [40]. Since we want $Y_\nu \sim Y_e$, the electroweak seesaw form of the neutrino mass matrix is important.

We find the effective mass matrix to be of the ususal Type I seesaw form

$$m_{eff} = -mM_{\chi_R^0}^{-1}m^\top \tag{205}$$

and the diagonalization of this matrix gives the light neutrino masses. The masses squared are most relevant since the mass squared difference is the observable in experiments, so we define the mass squared matrix to be $H = m_{eff}^\dagger m_{eff}$. The eigenvalues of H are thus the masses squared [58]:

$$m_1^2 = \frac{a}{3} - \frac{1}{3}p\left(\cos\phi + \sqrt{3}\sin\phi\right) \tag{206}$$

$$m_2^2 = \frac{a}{3} - \frac{1}{3}p\left(\cos\phi - \sqrt{3}\sin\phi\right) \tag{207}$$

$$m_3^2 = \frac{a}{3} + \frac{2}{3}p\cos\phi \tag{208}$$

where

$$
\begin{aligned}
p &= \sqrt{a^2 - 3b} \\
\phi &= \frac{1}{3}\arccos\left(\frac{1}{p^2}\left(a^3 - \frac{9}{2}ab + \frac{27}{2}c\right)\right) \\
a &= \mathrm{Tr}(H) \\
b &= H_{11}H_{22} + H_{11}H_{33} + H_{22}H_{33} - H_{12}^2 - H_{13}^2 - H_{23}^2 \\
c &= \mathrm{det}H.
\end{aligned} \tag{209}
$$

In order to get correct neutrino results, we require that $m_1^2 < m_2^2 < m_3^2$. With this restriction, the neutrino mass spectrum can have either normal or inverted ordering [58]:

- Normal ordering ($m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$):

$$
\begin{aligned}
m_{\nu_1}^2 &= m_1^2 \qquad m_{\nu_2}^2 = m_2^2 \qquad m_{\nu_3}^2 = m_3^2 \\
\Delta m_{12}^2 &= \frac{2}{\sqrt{3}} p \sin\phi > 0 \\
\Delta m_{13}^2 &= p\left(\cos\phi + \frac{1}{\sqrt{3}}\sin\phi\right) > 0
\end{aligned}
$$

- Inverted ordering ($m_{\nu_3} < m_{\nu_2} < m_{\nu_1}$):

$$
\begin{aligned}
m_{\nu_3}^2 &= m_1^2 \qquad m_{\nu_1}^2 = m_2^2 \qquad m_{\nu_2}^2 = m_3^2 \\
\Delta m_{12}^2 &= p\left(\cos\phi - \frac{1}{\sqrt{3}}\sin\phi\right) > 0 \\
\Delta m_{13}^2 &= -p\left(\cos\phi + \frac{1}{\sqrt{3}}\sin\phi\right) < 0
\end{aligned}
$$

We can also find expressions for the mixing angles from the eigenvectors of H. After normalizing the eigenvectors,

$$\sin\theta_{13} = |\,(U_\nu)_{13}\,| \tag{210}$$

$$\sin\theta_{23} = \frac{|\,(U_\nu)_{23}\,|}{\sqrt{1 - |\,(U_\nu)_{13}\,|^2}} \tag{211}$$

$$\sin\theta_{12} = \frac{|\,(U_\nu)_{12}\,|}{\sqrt{1 - |\,(U_\nu)_{13}\,|^2}} \tag{212}$$

where $(U_\nu)_{ij}$ are the components of the eigenvalues and are functions of H elements and m_ν^2 [58]. Since we know θ_{13} is small (but not zero), $(U_\nu)_{13}$ is also small; therefore the denominator for $\sin\theta_{23}$ and $\sin\theta_{12}$ is close to one. From this, we determine that $(U_\nu)_{23} \sim \frac{1}{\sqrt{2}}$ and $(U_\nu)_{12} \sim \frac{1}{2}$.

To examine more precisely the value of the neutrino mass and mixings in this theory, we must assign specific values to the free parameters. These parameters are

$$\lambda_i, \kappa_{iii}, \tan\beta, \nu_1, \nu_3, \nu_i^c, A_{\lambda_i}, A_{\kappa_{iii}}, A_{\nu_{ii}} \tag{213}$$

under the MFV assumption, where we define $\nu_1 = \nu_2 \neq \nu_3$ in order to get the correct neutrino mass hierarchy [23,58]. Choosing the parameters values carefully, we find that this theory can account for the observed neutrino masses and mixings.

6 Phenomenology of R-Parity

With all the different R-parity violating theories we have discussed, experimental results will reveal which one is most accurate. Each theory predicts the correct neutrino masses and mixings, so the differentiating factor will be the decay of the lightest supersymmetric particle. Recall that R-parity violation requires that the LSP is *not* stable, as it is in the MSSM, but the exact form and branching ratio can vary from theory to theory. However, we will mention one case where an R-parity violating theory resembles the MSSM, necessitating careful analysis of future results. Thus far the LHC and other experiments have not observed any signs of supersymmetry, but the $\not{R}_P$ theories can account for this null result. The current LHC results do place some restrictions on $\not{R}_p$ supersymmetry couplings, which we will briefly discuss in the last section.

6.1 Collider Signatures of $\not{R}_p$ Theories

As discussed previously, one of the major predictions of R-parity violating theories is the decay of the lightest supersymmetric particle, the LSP. In collider experiments we want to observe decays that result in Standard Model particles in distinct combinations or energies. Various supersymmetric particles can be the LSP, as shown in Table 9, but the neutralino generates the most exotic signals [49]. The products of neutralino decays are listed in Table 10. The lifetime of the LSP depends on the particle content of the supersymmetric theory and the value of the $\not{R}_p$ couplings, and since neutrino experiments point to small couplings the LSP lifetime is long, leading to displaced vertices. Physically, displaced vertices are decays that occur up to tens of centimeters from the orginal pp interaction inside the detector. Both vertices of this type and the particles produced in the decay can be signatures of physics beyond the Standard Model.

LSP	Decays
$\tilde{t}_1$	$t\bar{\nu}, j\bar{\nu}, be_i^+, je_i^+$
$\tilde{b}_1$	$b\bar{\nu}, j\bar{\nu}, te_i^-, je_i^-$
$\tilde{\chi}_1^0$	$e_i^\pm W, \nu Z, e_i^\pm H^\mp, \nu H_i^0$
$\tilde{\chi}_1^\pm$	$e_i^\pm Z, \nu W^\pm, e_i^\pm H_i^0, \nu H_i^0$
$\tilde{\tau}^\pm$	$e_i^\pm \nu, \bar{q}q', hW^\pm$
$\tilde{\nu}_3$	$\bar{q}q, \bar{e}_i e_j, WW, ZZ, hh, HH$

Table 9: Possible LSPs and their main decay products [49].

Name of Decay	Decay Products
Leptonic: $\ell\ell$	$\nu\ell^+\ell^-$ for $\ell = e, \mu$
Leptonic: $\tau\tau$	$\nu\tau^+\tau^-$
Leptonic: $\tau\ell$	$\tau\nu\ell$
Semi-leptonic: jj	$\nu\bar{q}q$
Semi-leptonic: τjj	$\tau\bar{q}q'$
Semi-leptonic: ℓjj	$\ell\bar{q}q'$
Semi-leptonic: bb	$\nu\bar{b}b$
Invisible	$\nu\nu\nu$

Table 10: The main decays of $\tilde{\chi}_1^0$ [12].

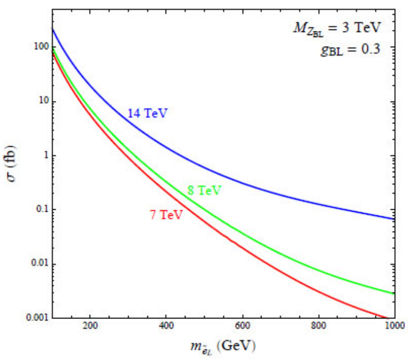
6.1.1 Minimal $U(1)_{B-L}$ Theory

In the minimal $U(1)_{B-L}$ theory we introduced and broke a new symmetry, replacing R-parity and leading to the Z'-boson. This particle can be used in slepton production, which then decays as follows [49]:

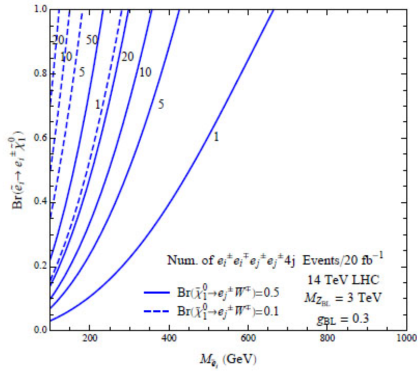
$$pp \to \gamma, Z, Z' \to \tilde{e}_i^+ \tilde{e}_i^- \to e_i^\pm \tilde{\chi}_1^0 e_i^\mp \tilde{\chi}_1^0 \to e_i^\pm e_i^\mp e_j^\pm e_k^\pm W^\pm W^\mp. \tag{214}$$

After the W-bosons decay into hadrons (producing four jets), we are left with four leptons, three of one charge and one of opposite charge.

First, we estimate the cross-section for the first decay of the chain, namely $pp \to \gamma, Z, Z' \to \tilde{e}_i^+ \tilde{e}_i^-$. Assuming the mass of Z'-boson is 3 TeV and $g_{B-L} = 0.3$, the cross-section for this decay as a function of $m_{\tilde{e}}$ is found in Figure 13a [49]. The decay is most likely when the mass of the selectron is small. With this information in place, we can now look at the decay product: the four leptons. The branching ratio of the decay can be calculated based on the cross-section results and are also dependent on $m_{\tilde{e}}$. The number of expected events in the LHC (at $\sqrt{s} = 14$ TeV) is shown in Figure 13b. Regardless of the selectron mass or branching ratio, this



(a) The cross-section of $pp \to \gamma, Z, Z' \to \tilde{e}_i^+ \tilde{e}_i^-$ as a function of the slepton mass.



(b) The number of four lepton events at 14 TeV.

Figure 13: Predictions for the minimal $U(1)_{B-L}$ theory [49].

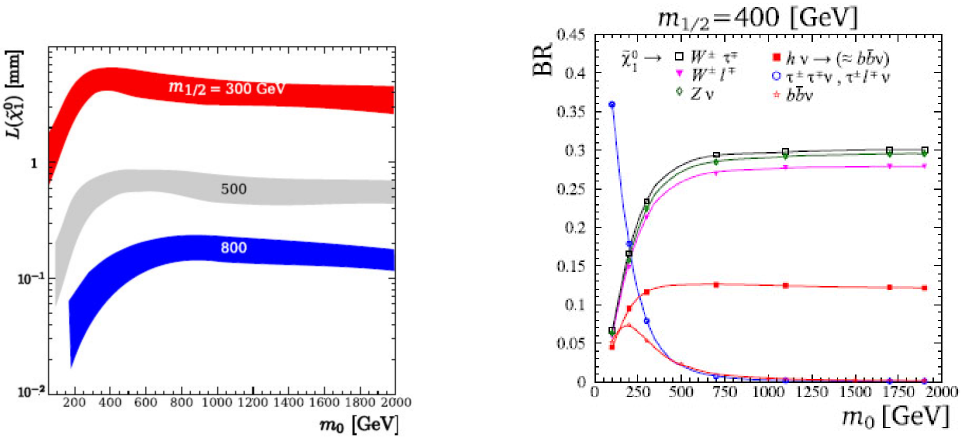
decay will be rare and therefore difficult to observe. However, at the upgraded 14 TeV LHC the expected luminosity is around 80 fb^{-1}, so the number of events indicated in Figure 13b will be multiplied by about four.

6.1.2 Bilinear $\not{R}_p$

The bilinear R-parity violation model does not contain the additional Z'-boson, but has similar restrictions as the $U(1)_{B-L}$ theory. The $\not{R}_p$ couplings must naturally still be small and the decaying LSP is still a major feature of the theory. In addition, the BRPV generates displaced vertices because the trilinear couplings are very small [12]. The BRPV, using supergravity to softly break supersymmetry, has eleven free parameters:

$$m_0, m_{1/2}, \tan\beta, \text{sign}(\mu), A_0, \epsilon_i, \Lambda_i \tag{215}$$

where m_0 is the gaugino mass, $m_{1/2}$ is the soft scalar supersymmetry-breaking mass, and A_0 is the trilinear term [12]. The parameters ϵ and Λ, which we discussed in Section 5.2.1, affect the decay length of the LSP but not the cross-section, as shown in Figure 14a [12]. Therefore, the cross-section of the LSP decay depends, as expected, primarily on m_0. Assuming $A_0 = -100$ GeV, $\tan\beta = 10$, positive μ, $m_{1/2} = 400$ GeV and that the LSP is a neutralino, the branching ratio of various decay channels can be found with the results in Figure 14b [12]. In it, ϵ and Λ have been fixed at optimal values to generate the most accurate neutrino oscillation results. For small m_0, the neutralino is most likely to take a decay path involving leptons, particularly τ, while heavier m_0 values generate decays involving quarks. This result is sensible as quarks are generally heavier than leptons and



(a) The decay length of the LSP as a function of the gaugino mass. The bands are limited to the correct neutrino values, and we note that this plot is valid for *any* ϵ and Λ.

(b) The LSP decay branching ratio for optimal values of ϵ and Λ. The different lines depict various decay channels.

Figure 14: Predictions for the BRPV model [12].

are therefore more likely to be produced in decays involving similarily heavy or energetic particles.

Because of the LSP decays, the popular method of looking for supersymmetry through missing traverse energy is weakened. The products of the LSP decay are Standard Model particles and so their energy will be accounted for in the collider, meaning that less of it is "missing." Searches for R-parity violating supersymmetry must then take a different approach, looking for distinct decay patterns and displaced vertices instead of missing energy.

6.2 Possible Similarities to the MSSM

A complication to the observation of R-parity violating theories is the fact that it may look very similar to the MSSM *without* $\not{R}_p$. One of the primary consequences of R-parity conservation is a stable LSP, but with R-parity violation the LSP can decay in such a way as to be invisible. We take the LSP to be a neutralino[22] $\tilde{\chi}^0$ which, in spontaneous $\not{R}_p$, can decay into the majoron J and neutrino as $\tilde{\chi}^0 \to J\nu$ [35]. This decay mode is invisible as the majoron is a component of the Z'-boson and the neutrino is indistinguishable from those from other sources. We will show that in a certain model this decay mode is the dominant one, thereby

[22]Recent results have eliminated the neutralino as a possible dark matter candidate, but the gravitino, axion or axino remain viable options in $\not{R}_p$ theories [35].

making the LSP appear as if it is stable.

In this model, we add three $SU(2)\times U(1)$ singlets to the MSSM: $\hat{\nu}^c$ with lepton number $L=-1$, $\hat{S}$ with $L=1$, and $\hat{\Phi}$ with $L=0$ [36]. The superpotential for this theory is

$$\begin{aligned} W =& Y_u^{ij}\hat{Q}_i\hat{u}_j\hat{H}_u + Y_d^{ij}\hat{Q}_i\hat{d}_j\hat{H}_d + Y_e^{ij}\hat{L}_i\hat{e}_j\hat{H}_d \\ &+ Y_\nu^i\hat{L}_i\hat{\nu}^c\hat{H}_u - Y_0\hat{H}_d\hat{H}_u\hat{\Phi} + Y\hat{\Phi}\hat{\nu}^c\hat{S} + \frac{\lambda}{3!}\hat{\Phi}^3 : \end{aligned} \tag{216}$$

the first line is the MSSM superpotential, the first term of the second line fixes lepton number, the second term generates a μ term, and the last two terms give mass to the new singlets once $\hat{\Phi}$ has a v.e.v. [35]. Note that this superpotential does not contain any terms with dimensions of mass, thereby avoiding the μ problem [35]. The theory also contains soft symmetry breaking terms:

$$\begin{aligned} \mathcal{L}_{soft} =& |Y\Phi\tilde{S} + Y_\nu^i\tilde{\nu}_iH_u + M_R\tilde{S}|^2 + |Y_0\Phi H_u + \hat{\mu}H_u|^2 + |Y\Phi\tilde{\nu}^c + M_R\tilde{\nu}^c|^2 \\ &+ |-Y_0\Phi H_d = \hat{\mu}H_d + Y_\nu^i\tilde{\nu}_i\tilde{\nu}^c|^2 + |-Y_0H_uH_d + Y\tilde{\nu}^c\tilde{S} - \delta^2 + M_\Phi\Phi + \frac{\lambda}{2}\Phi^2|^2 \\ &+ \ldots \end{aligned} \tag{217}$$

where the ellipsis represents terms summed over the gauge group indices i and α [36]. Lepton number is conserved in the superpotential terms, but after electroweak symmetry breaking, some particles gain v.e.v.s: the Higgs boson, sneutrinos, and new singlets $\langle\Phi\rangle = v_\Phi/\sqrt{2}$, $\langle\tilde{\nu}^c\rangle = v_R/\sqrt{2}$, and $\langle\tilde{S}\rangle = v_S/\sqrt{2}$ [35]. As before, v_R, v_L and now v_S violate lepton number and R-parity, and with R-parity broken Majorana neutrino masses are generated. The effective mass matrix is

$$\left(m_\nu^{eff}\right) = a\Lambda_i\Lambda_j + b\left(\epsilon_i\Lambda_j + \epsilon_j\Lambda_i\right) + c\epsilon_i\epsilon_j \tag{218}$$

where $\Lambda_i = \epsilon v_d + \mu v_L$, $\epsilon_i = Y_\nu v_R/\sqrt{2}$, $\mu = \hat{\mu} + Y_0 v_\Phi/\sqrt{2}$ and a, b, and c depend on the v.e.v.s, Y_0 and the fermion mass matrix [33]. From neutrino physics, the parameters Λ and ϵ are constrained; specifically, Λ_i/m_{SUSY}^2 and $|\epsilon_i/\mu|$ are small [35]. With these requirements as well as imposing $v_L \ll lv_R$ and $v_L/V < 1$, we find an expression for the majoron:

$$J \simeq \left(\frac{-v_dv_L^2}{Vv^2}, \frac{v_uv_L^2}{Vv^2}, \frac{v_{L_1}^2}{V}, \frac{v_{L_2}^2}{V}, \frac{v_{L_3}^2}{V}, 0, \frac{v_s^2}{V}, \frac{-v_R^2}{V}\right) \tag{219}$$

where $V^2 = v_S^2 + v_R^2$ and J is in the basis of Im $\left(H_d^0, H_u^0, \tilde{\nu}_1, \tilde{\nu}_2, \tilde{\nu}_3, \Phi, \tilde{S}, \tilde{\nu}^c\right)$ [35]. The important result here is that the majoron is a gauge singlet.

We now combine the information about the neutrinos and majoron to discuss

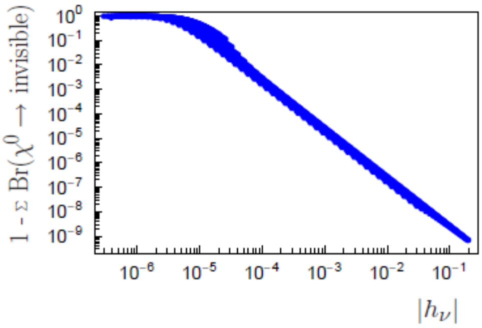


Figure 15: Plot of the visible decay modes of the neutralino as a function of $|h^\nu|$ [35].

their interaction. In general this coupling is quite complicated, but in the limit where $v_R, v_S \to \infty$ the approximate form is

$$C_{\tilde{\chi}^0 \nu_k J} \simeq \frac{U_\nu^\top \cdot \vec{\epsilon}}{V} N_{14} + \frac{U_\nu^\top \cdot \vec{\nu}_L}{V} \left(g' N_{11} - g N_{12}\right) + \ldots \tag{220}$$

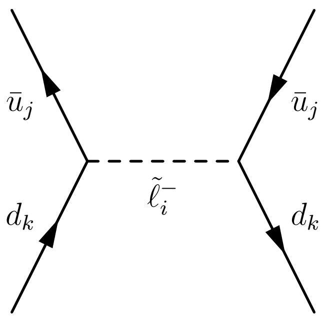
where U_ν is the matrix that diagonalizes the neutrino mass matrix and the Ns are coefficients [35]. Of the three singlets, only $\hat{\nu}^c$ is required to be present; if $\hat{S}$ is absent, the coupling behaves in the same way. From Equations 218 and 220, we see that the most important parameters are $|Y^\nu| = \sqrt{\sum_i \left(h_\nu^i\right)^2}$ and v_R [35]. Therefore, the plot in Figure 15 shows the sum over the neutralino decay branching ratios that contain a visible particle as a function of $|Y^\nu| = |h^\nu|$. The plot shows that, out of the expected 2.5×10^7 neutralinos at the LHC, only 100 will decay visibly for $|h^\nu| \leq 2.5 \times 10^{-3}$ [35]. In addition the invisible decay width of the neutralino does not depend on any MSSM parameters [35]. As a result, it will appear that the neutralino does not decay, as is the case in the MSSM.

6.3 LHC Results

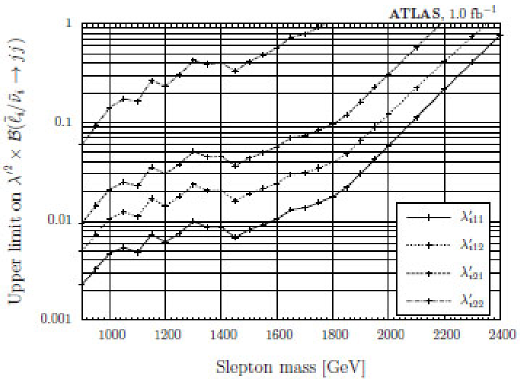
Though the LHC has not yielded any results beyond the expectations of the Standard Model, we can place bounds on the $\not{R}_p$ couplings λ and λ', which come from various decay channels.

A general area of experimental interest is the production and decay of sleptons into Standard Model particles. In this section, we will look at sleptons produced by the $\lambda'_{ijk} L_i Q_j \bar{D}_k$ term. These sparticles are formed, via this term, by

$$\bar{u}_j + d_k \to \bar{\ell}_i^- \tag{221}$$



(a) Diagram for dijet production.



(b) The upper bounds given by the ATLAS experiment on λ'_{1jk} couplings [21].

Figure 16: Diagram and results for the slepton dijet decay channel.

and can then decay by the same term as

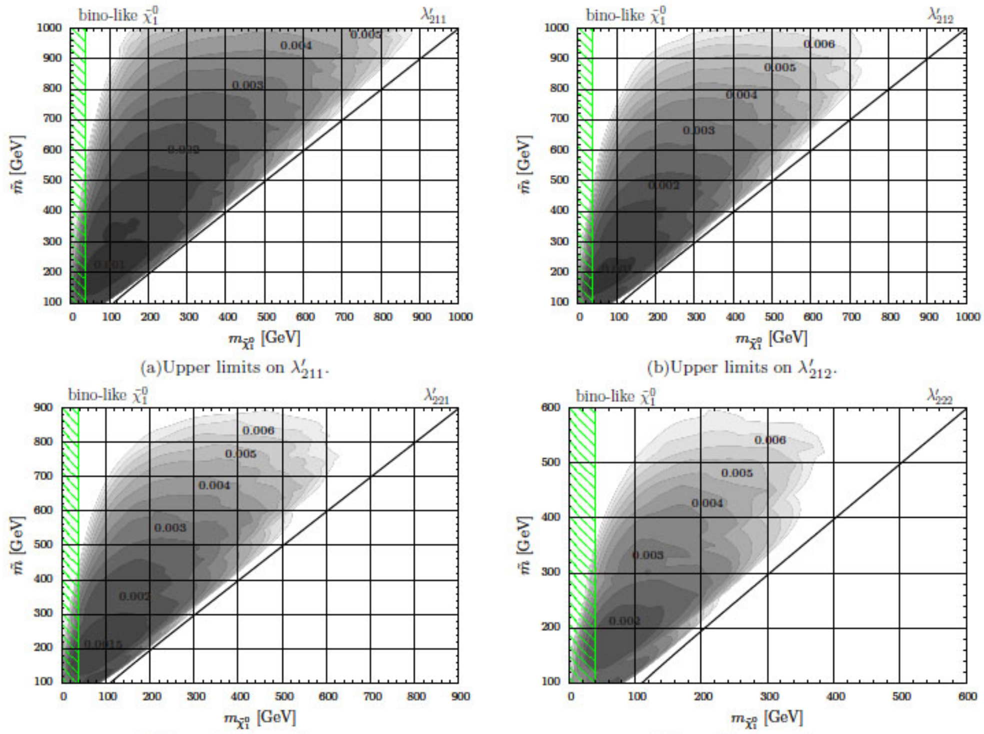
$$\tilde{\ell}_i^- \to \bar{u}_j d_k \tag{222}$$

or to neutralinos and charginos by

$$\tilde{\ell}_i^- \to \begin{cases} \ell_i^- \chi_j^0 \\ \nu_i \chi_j^- \end{cases} ; \tag{223}$$

note that sleptons can be produced and decay via other channels as well [21]. Equation 221 implies that any combination of quarks can produce sleptons, but we only consider coupling to the first two generations ($j, k = 1, 2$) as the top and bottom quarks are too heavy. The cross-sections of these processes depend on which quarks are involved (that is, which coupling λ') and the mass of the slepton that is produced. Predictably, the heavier the slepton, the smaller the cross-section.

The decay channels of the slepton are slightly more complex. The decay via the $\not{R}_p$ term leads to the production of two jets, as pictured in Figure 16a. Since this process depends on the same term, we get constraints on the same coupling constant λ'. If we estimate the slepton mass $m_{\tilde{\ell}} = 500$ GeV and $\lambda' = 0.05$, we get a decay width of $\Gamma(\tilde{\ell}_i^- \to \bar{u}_j d_k) \approx 75$ MeV, which leads to a narrow resonance in the invariant mass spectrum of the dijets [21]. However, we need a $m_{\tilde{\ell}} > 1$ TeV in order to have this process be visible amidst the large QCD background at the LHC [21]. Therefore, the plot in Figure 16b shows the dependence of the coupling λ'_{1jk} $\times$branching ratio of dijet production on the slepton mass from 1 TeV upward.



(a)Upper limits on λ'_{211}.

(b)Upper limits on λ'_{212}.

(c)Upper limits on λ'_{221}.

(d)Upper limits on λ'_{222}.

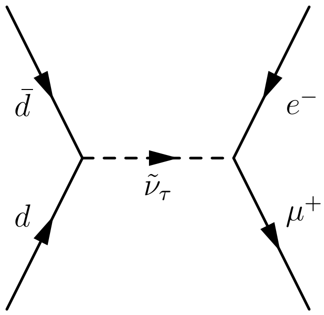
Figure 17: The upper bounds given by the ATLAS experiment on λ'_{1jk} couplings for a bino-like neutralino [21].

As seen in Equation 223, sleptons can also decay into neutralinos. These neutralinos can be either bino-like, wino-like, or higgsino-like; each option has a different mass hierarchy between the four neutralinos, μ, and $m_{\tilde{\ell}}$ [21]. The bino-like contribution is most significant in the MSSM, so we will focus on it. For bino-like neutralino models, we have[23] $M_2, \mu \gg M_1, m_{\tilde{\ell}}$ [21]. A major signature for neutralino decay is the presence of a muon pair $\mu\bar{\mu}$, both with the same sign.[24] The $\bar{\mu}^-$ is generated by the slepton production in Equation 221 while the μ^- is generated by the slepton decay in Equation 223 [21]. This decay places upper limits on the $\not{R}_p$ coupling λ'_{2jk} where $j, k = 1, 2$ [21]. Figure 17 shows these limits as a function of the slepton and neutralino masses. The coupling is therefore quite small, as the largest upper bound is only 0.006.

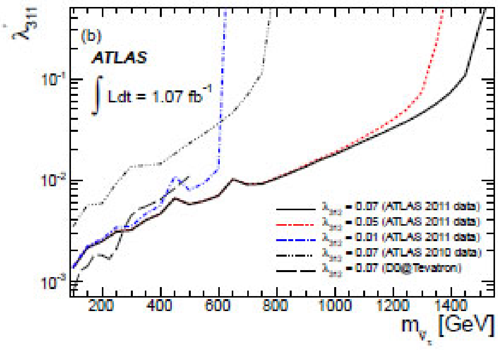
LHC searches have also explored the possibility of $e\mu$ resonance, which is the decay of the LSP $\tilde{\nu}_\tau$ to $e\mu$ as pictured in Figure 18a. All couplings except λ'_{311} and

[23] Recall that M_1 is the bino mass and M_2 is the wino mass.

[24] This muon pair is distinct as pair-produced muons will naturally be of opposite sign.



(a) Diagram for $e\mu$ resonance. The coupling λ'_{311} acts on the left-hand vertex.



(b) The upper bounds given by the ATLAS experiment on λ'_{1jk} couplings [37].

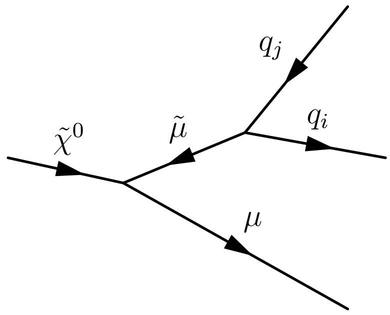
Figure 18: Diagram and results for λ'_{311}, imposed by $e\mu$ resonance.

λ_{312} are set to zero. The candidate e and μ particles have traverse momentum$>$ 25 GeV, pseudorapidity $|\eta| \lesssim 2.4$, are separated by $\sqrt{\Delta\eta^2 + \Delta\phi^2} > 0.2$, and have opposite charges [37]. No events of this type were observed at the LHC, which sets an upper limit on the product of the cross-section $\sigma(pp \to \tilde{\nu}_\tau)$ and $BR(\tilde{\nu}_\tau \to e\mu)$ [37]. In turn, this limit places a bound on the couplings as a function of $m_{\tilde{\nu}_\tau}$; this dependency of λ'_{311} on this mass and the other coupling λ_{312} is pictured in Figure 18b. From this graph, we can conclude that the mass of the LSP should not be more than about 1400 GeV at the maximum else the λ'_{311} coupling would be unphysically large.

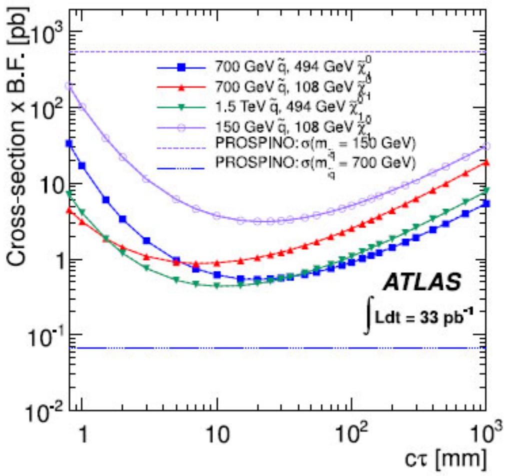
In addition to slepton channels, the LHC also examined displaced vertices. These vertices may be the result of an LSP decay due to a non-zero coupling λ'_{2ij} as in Figure 19a [37]. To look for these decays, several selection criteria are implemented but no vertices are observed to fit [37]. The null observation places restrictions on the cross-section$\times$branching fraction of the process, dependent on the neutralino and squark masses; this result is shown in Figure 19b [37]. Though no results beyond the expectation of the Standard Model have been observed, we are optimistic that future experiments at the LHC after its upgrade may yield results that support supersymmetry and R-parity violation.

7 Conclusion

In this paper, we have seen that the Standard Model is inadequate for explaining the neutrino mass that is predicted by the observation of neutrino oscillations. Supersymmetry is proposed to solve this and other problems, but some new issues arise, notably fast proton decay. In order to restrict the MSSM R-parity



(a) Diagram for displaced vertex with two jets. The coupling λ'_{2ij} acts on the right-hand vertex.



(b) The upper bounds given by the ATLAS experiment on the cross-section times the branching fraction as a function of the $\tilde{\chi}^0$ lifetime [37].

Figure 19: Diagram and results from displaced vertices searches.

is imposed, which has the consequence of conserving baryon and lepton number. However, for neutrino mass lepton number must be violated and so we abandon R-parity in favor of other methods of constraining the proton decay. We can either explicitly introduce R-parity violating terms into the superpotential of the MSSM or generate these terms spontaneously by breaking a new $U(1)$ symmetry. The new terms are either bilinear or trilinear, and contribute to neutrino masses at either the tree or loop level. In most cases the bilinear tree-level contributions are most significant, but only generate mass for one neutrino; therefore the loop level contributions are small, but important. We examined the most basic R-parity violating theory, which only involves bilinear terms, but the introduction of flavor symmetries restrict the theory more adequately and offer an explanation for the presence of the R-parity violating terms. In addition, R-parity can be broken explicitly with the addition of new superfields *a la* the seesaw mechanism; this method provides accurate neutrino masses and mixings at the cost of adding new fields to the MSSM. All of these new theories have distinct collider signatures, and the hope is that we will observe one of them, offering more insight into physics beyond the Standard Model.

Acknowledgments

I would like to thank Professor Kelly Stelle for all his helpful advice over the summer. Though my dissertation has an unusual topic for a theoretical course, he encouraged me to pursue my interest.

References

[1] Aitchison, Ian J.R., 2005, *Supersymmetry and the MSSM: An Elementary Introduction*, arXiv:0505105v1 [hep-ph].

[2] Allanach, B.C. and M.A. Bernhard, 2012, *Including R-parity violation in the numerical computation of the spectrum of the minimal supersymmetric standard model: SOFTSUSY3.3.3*, arXiv:0903.1805v7 [hep-ph].

[3] D'Ambrosio, G., G.F. Giudice, G. Isidori, and A. Strumia, 2002, *Minimal Flavour Violation: an effective field theory approach*, arXiv:0207036v2 [hep-ph].

[4] Antoniadis, I., E. Dudas, D.M. Ghilencea, and P. Tziveloglou, 2008, *MSSM with Dimension-five Operators*, arXiv:0806.3778v2 [hep-ph].

[5] Bahcall, J.N. and H.A. Bethe, 1993, "Do Solar Neutrino Experiments Imply New Physics?", *Phys. Rev. D* **47**, pp. 1298-1301.

[6] Barbier, R., et. al. 2005, *R-Parity-Violating Supersymmetry*, arXiv:0406039v2 [hep-ph].

[7] Barger, Vernon, Pavel Fileviez Pérez, and Sogee Spinner, 2009, *Minimal Gauged $U(1)_{B-L}$ Model with Spontaneous R-Parity Violation*, arXiv:0812.3661v2 [hep-ph].

[8] Barr, S. M., 2004, "New Type of Seesaw Mechanism for Neutrino Mass", *Phys. Rev. Letters* **92**, 101601.

[9] Bazzocchi, F., S. Morisi, E. Peinado, J.W.F. Valle, and A. Vicente, 2013, *Bilinear R-parity violation with flavor symmetry*, arXiv:1202.1529v2 [hep-ph].

[10] Beck, Mark, 2012, *Quantum Mechanics: Theory and Experiment*, pp. 273-276.

[11] Burgess, Cliff and Guy Moore, 2007, *The Standard Model: A Primer*, Cambridge University Press.

[12] de Campos, F., et.al., 2008, *Probing bilinear R-parity violating supergravity at the LHC*, arXiv:0712.2156v2 [hep-ph].

[13] Chang, Xue, Chun Liu, and Yi-Lei Tang, 2013, *Phenomenological Aspects of R-parity Violating Supersymmetry with A Vector-like Extra Generation*, arXiv:1303.7055v2 [hep-ph].

[14] Cheng, Taoli, et. al., 2012, *Toward the Natural and Realistic NMSSM with and without R-Parity*, arXiv:1207.6392v2 [hep-ph].

[15] Chun, Eung Jin and Seong Chan Park, 2004, *Neutrino Mass from R-parity Violation in Split Supersymmetry*, arXiv:/0410242v2 [hep-ph].

[16] Close, Frank, 2010, *Neutrino*, Oxford University Press.

[17] Cottin, Giovanna, Marco Aurelio Diaz, Sebastián Olivares, and Nicolás Rojas, 20012, *Neutrinos and the Higgs Boson in Split Supersymmetry*, arXiv:1211.1000v2 [hep-ph].

[18] Csáki, Csaba, Yuval Grossman and Ben Heidenreich, 2012, *MFV SUSY: A Natural Theory for R-parity Violation*, arXiv:1111.1239v3 [hep-ph].

[19] Diaz, Marco Aurelio, 2005, *Neutrinos in Supersymmetry*, arXiv:0507044v1 [hep-ph].

[20] Diaz, Marco Aurelio, Pavel Fileviez Pérez, and Clemencia Mora, 2009, *Neutrino Masses in Split Supersymmetry*, arXiv:0605285v5 [hep-ph].

[21] Dreiner, H.K. and T. Stefaniak, 2012, *Bounds on R-parity Violation from Resonant Slepton Production at the LHC*, arXiv:1201.5014v2 [hep-ph].

[22] Ellwanger, Ulrich, Cyril Hugonie, and Ana M. Teixeira, 2011, *The Next-to-Minimal Supersymmetric Standard Model*, arXiv:0910.1785v5 [hep-ph].

[23] Escudero, Nicolás, Daniel E. López-Fogilani, Carlos Muñoz, and Roberto Ruiz de Austri, 2008, *Analysis of the parameter space and spectrum of the $\mu\nu$SSM*, arXiv:0810.1507v2 [hep-ph].

[24] Faessler, Amand, 2012, *The Neutrinoless Double Beta Decay, Physics beyond the Standard Model and the Neutrino Mass*, arXiv:1203.3648v1 [hep-ph].

[25] Feng, Jonathan L., 2013, *Naturalness and the Status of Supersymmetry*, arXiv:1302.6587v2 [hep-ph].

[26] Figueroa-O'Farrill, José Miguel, 2001, *Busstepp Lectures on Supersymmetry*, arXiv:0109172v1 [hep-th].

[27] Ghosh, Pradipta and Sourov Roy, 2009, *Neutrino masses and mixing, lightest neutralino decays and a solution to the μ problem in supersymmetry*, arXiv:0812.0084v2 [hep-ph].

[28] Gonzalez-Garcia, M.C., 2008, "Neutrinos: Theory", *The Dawn of the LHC Era: TASI 2008*. pp. 527-564.

[29] Griffiths, David, 2008, *Introduction to Elementary Particles*, 2nd revised edition, WILEY-VCH Verlag GmbH & Co.

[30] Grossman, Yuval and Subhendu Rakshit, 2004, *Neutrino masses in R-parity violating supersymmetric models*, arXiv:0311310v3 [hep-ph].

[31] Hegedus, Kayla, 2012, *An Introduction to Neutrino Oscillations*, Thesis, Whitman College.

[32] Hirsch, M., and J.W.F. Valle, 2004, *Supersymmetric Origin of Neutrino Mass*, arXiv:0405015v1 [hep-ph].

[33] Hirsch, M., J.C. Romao, J.W.F. Valle, and A. Villanova del Moral, 2004, *Invisible Higgs Boson Decays in Spontaneously Broken R-Parity*, arXiv:0407269v2 [hep-ph].

[34] Hirsch, Martin, 2006, *Phenomenology of neutrinoless double beta decay*, arXiv:0609146v1 [hep-ph].

[35] Hirsch, M. and W. Porod, 2006, *R-parity violation: Hide & Seek*, arXiv:0606061v3 [hep-ph].

[36] Hirsch, M., J.C. Romao, J.W.F. Valle, and A. Villanova del Moral, 2006, *Production and decays of supersymmetric Higgs bosons in spontaneously broken R-parity*, arXiv:0512257v2 [hep-ph].

[37] Jackson, Paul D., on behalf of the ATLAS Collaboration, 2011, *Search for R-parity violating supersymmetry with the ATLAS detector*, arXiv:1112.0369v1 [hep-ex]

[38] Klauber, Robert D., 2012, "The Seesaw Mechanism", *Pedagogic Aids to Quantum Field Theory*, `http://www.quantumfieldtheory.info/`.

[39] Kobakhidze, Archil, Nadine Pesor, and Raymond R. Volkas, 2010, *Neutrino masses and sparticle spectra from stochastic superspace*, arXiv:1003.4782v2 [hep-ph].

[40] López-Fogilani, D.E. and C. Muñoz 2006, *Proposal for a Supersymmetric Standard Model*, arXiv:0508297v3 [hep-ph].

[41] Ma, Ernest, 2009, *Neutrino Mass: Mechanisms and Models*, arXiv:0905.0221v1 [hep-ph].

[42] Martin, Stephen P., 2008, *A Supersymmetry Primer*, arXiv:9709356v5 [hep-ph].

[43] Muñoz, Carlos, 2009, *Phenomenology of a New Supersymmetric Standard Model: the $\mu\nu$SSM*, arXiv:0909.5140v1 [hep-ph].

[44] Nakamura, K., and S.T. Petcov, May 2010, *Neutrino Mass, Mixing, and Oscillations*, JPG 37, 075021.

[45] Paradisi, Paride and David M. Straub, 2010, *The SUSY CP Problem and the MFV Principle*, arXiv:0906.4551v2 [hep-ph].

[46] Peinado, E. and A. Vicente, 2012, *Neutrino masses from R-parity violation with a Z_3 symmetry*, arXiv:1207.6641v2 [hep-ph].

[47] Pérez, Pavel Fileviez and Sogee Spinner, 2010, *The Fate of R-Parity*, arXiv:1005.4930v1 [hep-ph].

[48] Pérez, Pavel Fileviez, 2013, *On the Origin of R-parity Violation in Supersymmetry*, arXiv:1305.6935v2 [hep-ph].

[49] Pérez, Pavel Fileviez and Sogee Spinner, 2013, *Supersymmetry at the LHC and The Theory of R-parity*, arXiv:1308.0524v1 [hep-ph].

[50] Peskin, Michael E. and Daniel V. Schroeder, 1995, *An Introduction to Quantum Field Theory*, Westview Press.

[51] Polonsky, Nir, 1999, *The μ-Parameter of Supersymmetry*, arXiv:9911329v1 [hep-ph].

[52] Rakshit, Subhendu, 2004, *Neutrino masses and R-parity violation*, arXiv:0406168v1 [hep-ph].

[53] Randall, Lisa, 2005, *Warped Passages: Unraveling the Mysteries of the Universe's Hidden Dimensions*, New York: Harper Perennial.

[54] Randall, Lisa and Matthew Reece, 2013, *Single-Scale Natural SUSY*, arXiv:1206.6540v2 [hep-ph].

[55] Solomey, Nickolas, 1997, *The Elusive Neutrino: A Subatomic Detective Story*, New York: Scientific American Library.

[56] Valle, J.W.F., 2009, *Neutrino mass in supersymmetry*, arXiv:0911.3103v1 [hep-ph].

[57] Wess, Julius and Jonathan Bagger, 1992, *Supersymmetry and Supergravity*, second edition, Princeton University Press.

[58] Zhang, Hai-Bin, Tai-Fu Feng, Li-Na Kou, and Shu-Min Zhao, 2013, *Neutrino mixing in the $\mu\nu$SSM*, arXiv:1307.6284v2 [hep-ph].